THE MATHEMATICS STUDENT

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THIRTEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

The Indian Mathematical Society met for its Thirteenth Conference at Annamalainagar, under the auspices of the Annamalai University, on the 28th, 29th and 30th of December 1943.

The Conference opened at 12 A. M. (New Standard Time) on Tuesday the 28th December 1943 in the Srinivasa Sastri Hall. A message from Dr, Rajah Sir S. Rm. M. Annamalai Chettiar, Kt., LL.D., D.Lit., Founder Pro-Chancellor, Annamalai University, wishing success to the Conference was communicated by M. Ruthnaswami Esq., M.A., C.I E., Barrister-at-law, Vice-Chancellor of the Annamalai University, who welcomed the delegates and declared the conference open.

Mr. S. Mahadevan (Madras), Secretary of the Society, then read the Report of the Society's activities, and this was followed by the Presidential Address by Professor F. W. Levi, President of the Indian Mathematical Society.

Besides the reading of papers there were symposia on "Partially Ordered Sets", "Topology" and Modern Algebra and the Theory of Elementary Particles", the first two under the Chairmanship of Dr. R. Vaidyanathaswamy (Madras), and the third under the Chairmanship of Dr. B. S. Madhava Rao (Bangalore).

There was a popular evening lecture by Prof. A. A. Krishnaswami Aiyangar (Mysore) on "Random Numbers and Lines" on the 28th December followed by an interesting discussion on the logical implications of randomness. The next evening Mr. D. R. Kaprekar (Devlali) addressed the students of the Mathematics Association of the Annamalai University on "Demlo Numbers". The lecture was very much appreciated by the students and the delegates.

As in previous Conferences, there was a business meeting of the Society (at 9-30 P. M. on the 29th December) and a discussion on the teaching of Mathematics in Schools and Colleges (between 4-40 P. M. and 6 P. M. on the 30th). On the evening of the 28th there was a demonstration by Mr. L. Sanjiviraya, a blind calculator from the Cudappah district of his marvellous ability to do long calculations mentally. The visitors were taken to the temple at Chidambaram on the morning of the 30th December.

While the serious side of the Conference thus received due attention, the social side was by no means neglected. The guests were comfortably lodged in the Guest Houses and all the expenses connected with their boarding were

generously met by the Founder Pro-Chancellor, who also invited them to a garden party on the evening of the opening day. On the second evening the Vice-Chancellor of the Annamalai University was the host at another garden party. All the visitors and members of the University teaching staff were invited to both the parties.

A note-worthy feature of the Annamalainagar Conference was the "Mathematics Exposition" opened by Mr. S. V. Ramamurty, M.A., I.C.S., Advisor to His Excellency the Governor of Madras, and a long standing member of the Society, at 3–30 P. M. on the 28th December. The Exposition covered eleven rooms and contained over a thousand items relating to results of interest in Pure Mathematics and the rich and varied applications of Mathematics to modern life. The exposition attracted a large number of visitors from the neighbourhood, the room devoted to puzzles and recreations being particularly crowded with visitors.

During the course of the session the first prize problem for the award of the "Narasinga Rao Medal for Mathematical Research" was announced. The details of the problem and the conditions of award are given later in this issue.

Reception Committee

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Opening Speech

BY

M. RUTNASWAMI Esq., Vice Chancellor, Annamalai University.

Mr. President, Delegates to the Thirteenth Indian Mathematical Conference, Ladies and Gentlemen,

Allow me, on behalf of the University, its students, its teachers and authorities to offer you a hearty welcome to Annamalainagar. Here is no great city with the attractions and amenities of an up-to-date civilization. We have no museums, art galleries, hotels, restaurants, theatres and cinema houses, with which we could entertain you. Here we have only an ancient temple town on the one side of the railway station and a modern temple of learning on the other. We can offer you only the homely hospitality of a family of scholars, whose living is plain and whose thinking aspires to be high. And although it is I that say it, you will find nowhere a more devoted band of students of mathematics devoting hard won hours from the routine of teaching to the disinterested work of research in mathematics.

But you will pardon me Mr. President, if I as a lay and distant worshipper at the shrine of mathematics and as one concerned more at present with the administration of departments of learning rather than with any teaching, if I venture to ask the Conference to devote some attention to the methods of teaching Mathematics. I have sad memories of mathematical instruction in my youth. In the very first class of my High School, I had a Mathematics teacher who believed in putting mathematics into my head with a cane in the hand and a body and face vibrating with anger. If I have not taken kindly to mathematics, native obtuseness may not be the only reason. May I

therefore plead that Mathematics be presented to the young student not merely as an instrument of mental training but, as a recent English report on education has put it, as an instrument of material conquests and of social organization. Mathematics like many other sciences, arose from the necessities of primitive practical affairs as counting, the measurement of time, the determination of directions and of the needs of the builder and the sailor. Mathematics has flourished throughout its past from its touch with the business of every day life. And if at every stage of mathematical instruction, the pupil were made to realize that every part of his mathematical studies arises from, is related to, and is useful to this or that part of daily life, much greater interest in mathematical studies would be evinced than I am afraid obtains at present. To this end also the suggestion of the Report I have referred to already may be recommended to teachers and organizers of mathematical studies "that there should be no hard and fast separation between the different branches of mathematics like arithmetic, algebra and geometry each with its own technical apparatus and methods, but that mathematics should be taught as a science in which the topics are chosen so as to develop a grasp of mathematical ideas and these topics should be suggested and introduced by the examination of practical questions which have been of interest and utility to man in his affairs." One way of reaching this end, the Report suggests, is to use the problem method as the standard method of procedure in all early mathematical teaching. If mathematical ideas and rules are developed in this historical sequence and practical setting, may we not hope that these ideas and rules will acquire a human value and meaning for pupils, and that pupils will acquire that sense of the purposefulness of mathematics without which it may so easily become dead sea fruit.

I must apologise Mr. President for introducing these observations of an outsider into your deliberations. But here, as in other matters, an onlooker may see more of the game than those that play it.

In conclusion I have to convey to the President and Delegates of this conference the profound regret of the Patron of this Conference, the Rajah Sir Annamalai Chettiar, the Founder and Pro-Chancellor of this University for his inability to be present, and welcome you to the conference and open it. In his unavoidable absence, allow me to convey to you his message of cordial welcome to all the delegates assembled here and his best wishes for the success of the conference.

With that message I have the honour and the pleasure to declare the Conference open.

Secretary's Report on the working of the Society for the years 1942 and 1943

BY

S. MAHADEVAN, Secretary, *Madras*.

On behalf of the Managing Committee of the Indian Mathematical Society, allow me, Sir, to thank you and the other authorities of the Annamalai University for having so kindly invited us to hold our Conference in this ancient city. We are all the more happy that we are meeting under the auspices of an institution which owes its existence to the generosity and foresight of the great philanthropist, whose name it bears.

I regret very much to report during these two years the death of Messrs. Kareghat, I. C. S., and Professor Kashikar of the Wilson College, Bombay. They were both of them members of the Society for a long time and had done useful work in their own spheres.

The credit of being the Founder of our Society which was established in 1907 for the promotion of mathematical study and research in this country goes to the late Mr. V. Ramaswami Aiyar. During these 36 years the Society has been doing good work in spite of financial and other difficulties.

The idea of holding conferences of the Society to meet at different parts of the country originated with the Founder, and our first Conference was held in Madras in 1916. Since then we have been holding Conferences every two years and met successively in Bombay, Lahore, Poona, Bangalore, Nagpur, Trivandram, Delhi, Lucknow, Hyderabad and Aligarh. We are now holding the Thirteenth Session under the auspices of this unique University, thanks to the munificence and the active co-operation of the Founder Pro-Chancellor Raja Sir Annamalai Chettiar, KT., LL. D., and the Vice-Chancellor M. Ruthnaswami Esq., Barrister-at-law, C. I. E.

We met under the shadow of the war in the east at Aligarh, and now the war seems to be at our doors. The tense situation made us hesitate for a time whether we should not be wiser to postpone the Conference. It does a great credit to Prof. A. Narasinga Rao and his colleagues that, in spite of immense difficulties, they have not only organised this Conference but even gone a step further and arranged a Mathematics Exposition to be opened shortly by Mr. S. V. Ramamurty, one of the oldest and most distinguished members of our Society.

Besides bringing mathematicians together into fruitful contact at our biennial conferences, the Society maintains a well-equipped library of mathematical books and periodicals at our head-quarters in Poona and also arranges for the circulation of current periodicals. The Society also publishes

two quarterlies—The *Journal of the Indian Mathematical Society* and *The Mathematics Student.*

The *Journal* is conducted by a board of editors with Dr. R. Vaidyanathaswami of Madras as Chairman and Dr. C. N. Srinivasa Aiyangar of Bangalore as Secretary, and prints papers of an advanced character. During the period under report 61 papers were received for publication which were on the whole of a high standard and I wish to thank the referees for their willing co-operation in scrutinizing these papers.

The war and the economic situation are probably having their effect on mathematical research in India as there has been a definite fall in the number of papers received of late. Let us hope that this lull is only temporary and that this Conference will stimulate an increased output.

Recently we experienced great difficulty in getting paper and this led to the issue of a double number of the *Journal* for March and June. But thanks to our President, Prof. Levi, we have secured some paper for the *Journal* which has resumed its normal size and the authors are assured of their usual reprints in spite of increased cost of paper and printing.

The *Mathematics Student* is conducted by Prof. A. Narasinga Rao and a band of collaborators. It meets the needs of students and teachers of mathematics in colleges and of young research workers. It seeks to kindle enthusiasm in the minds of the students and to stimulate teachers to an improvement in their methods of teaching. I earnestly appeal to all teachers assembled here to co-operate in making this quarterly a means of improving the standards of mathematical teaching in this country. *The Mathematics Student* serves also as an information bureau on matters mathematical both in India and abroad. Teachers of Mathematics are requested to make this service as complete and efficient as possible by communicating all important items of information to the editor.

The Library is under the able management of Prof. Shintre of the Fergusson College, Poona. The war has hit us hard and we are not able to get journals published abroad regularly. It is hoped to get our file complete as soon as the war is over.

We are glad to announce that Prof. A. Narasinga Rao has given a donation of Rs. 1,000 to institute a gold medal for encouraging research in Mathematics. It is to be awarded to the best solution, or contribution towards the solution, of a specified problem in Mathematics which will be announced in *The Journal of the Indian Mathematical Society, The Mathematics Student* and the Press. A period of at least 18 months will be given for solving the problem. An expert committee will be appointed to select the problem and to adjudicate the award. The first Prize problem is under discussion and will be announced in the course of this session. We thank Prof. Narasinga

Rao for instituting this medal and hope that it may encourage the pursuit of research in this country.

Our Society wishes to expand its activities by undertaking the publication of monographs on select mathematical topics. All well-known Societies in the world undertake a similar kind of work, and we hope that with the financial co-operation of our Universities and Princes of India, we shall be enabled to undertake such a scheme.

In conclusion I wish to thank the various Universities for their unfailing support and for having sent exhibits and deputed delegates to this Conference and the local Government for the facilities given to their officers to attend the Conference.

MATHEMATICS EXPOSITION

Speech by Dr. A. Narasinga Rao, Local Secretary

requesting Mr. S. V. Ramamurty, to declare the Exposition open

Allow me to offer to you, Sir, and to all those who have gathered here in response to our invitation a most hearty and cordial welcome to this ancient city whose glories were sung all over the land at a time when the Roman Empire was crumbling to pieces.

In these days travelling is something more than a geometrical world line in the space-time continuum. It involves difficult problems of Applied Mathematics connected with the maintenance of equilibrium under periodic impulsive forces, of the coefficient of elasticity of the human body and of reasoning with railway officials working on postulates different from ours. That we should still have delegates from places as far away as Kapurthala, Lucknow and Calcutta, is to me, Sir, a very hopeful portent of the future of Mathematics in this country.

It is a matter of joy and pride for us, members of the Indian Mathematical Society, to have in our midst a fellow member of long standing who has won a reputation, rare even among the gifted men that constitute the Indian Civil Service, as an able administrator and a profound thinker. In the midst of the very onerous duties of your high office, it was very kind, nay, generous on your part to have agreed to open this exposition, and on behalf of the Indian Mathematical Society and the Annamalai University, I thank you most sincerely for it.

I should like particularly to express my delight at having in our midst Professor D. D. Kapadia who was Secretary of the Society jointly with Mr. V. Ramaswami Aiyar at its very foundation in 1907, and whom

neither age nor the necessity of being at Benares within two days has prevented from attending this session. I am also very glad to welcome into our midst Mr. A. T. Rajan, one of our two Senior Wranglers who is once more in our midst after a long service outside India.

I have been asked what Mathematics has to exhibit; whether the Mathematics Exhibition would begin with a blackboard and end with a piece of chalk? Is not Mathematics a search for that abstract structure which underlies a diversity of content in the texture of our experience, whether in the outer world of sense or the inner world of thought? Do not mathematicians create, like Brahma, worlds of their own with strange laws of their own choosing, juggling with them this way and that, till this solid world is lost in the mist of unreality? What can a subject whose votaries "do not know what they are talking about and do not care whether what they say is true" have to "expose" unless it be their own eccentricities which are already sufficiently well-known!

In reply I need only state that the abstract and the concrete are not opposites, but complements of each other and that in rising from the solid world of so-called reality into the heights of abstraction, a mathematician not only derives aesthetic satisfaction which such a process involves, but also returns with a more integral vision and a better appreciation of those relations which he was too near to notice. Who would have thought that the imaginary unit, the square root of -1 which we meet nowhere in the world of quantity, would help in giving the proper shape to our aeroplane propellers, or that four dimensional space time will help in predicting the motion of a planet round the Sun? The emergence of man, and his progress through the ages are closely linked with his ability to live in regions more and more abstract; and when at last he attempts to delve into the very heart of matter, his mathematics acquires the potency of a *mantra* which acts though using words whose meanings are not clearly understood.

In my opinion the purpose of an exhibition such as ours must be to show that reality is one and indivisible and that abstract mathematical formulae and concrete facts of experience are but different sides of reality, to drive away from mathematics the musty atmosphere of the lecture room and show that it is the efflorescence of the human spirit by its contact with the outer world of orderly relations and the aesthetic urge within. In judging this—our first attempt at a mathematics exposition, I look to you to apply not the standards of a critical mathematician but the generous and loving standards of fellow members of our Society.

Before requesting you, Sir, to declare this Exposition open, it is my duty to express my thanks to all those who have helped to make it possible. I have had it from far and near, from Annamalainagar to the U. S. A. whence we have obtained all our pictures from the Editor of Scripta Mathematica and books from Messrs. Simon and Schuster. I am greatly indebted to the

authorities of the Annamalai University, particularly to the Founder-Pro-Chancellor for having made this Conference possible and for their constant encouragement and help. Thanks to the engineer, the mechanic and my colleagues, a number of models have been quickly prepared. I must also thank the Universities of Madras, Mysore, Lucknow and Calcutta for their very kind co-operation, and individual helpers like Prof. A. A. Krishnaswmi Ayyangar, and Prof. B. S. Madhava Rao who have taken a lot of pains to prepare some very interesting exhibits. I have now great pleasure in requesting you, Sir, to open this Exposition.

Speech by Mr. S. V. Ramamurty, Adviser to H. E. Governor of Madras, declaring the Exposition open.

Let me thank the Indian Mathematical Conference for the honour it has done me in asking me to open this Mathematics Exposition.

It is indeed a happy idea to arrange this exhibition of things to expose the thought of mathematics. Thoughts and things are not apart from each other. They are different aspects of reality, yielded by analysis. Science and Philosophy have been popularly differentiated as studies of quantity and quality. There is need to perceive the common basis of quantity and quality. Long ago when I tried to reconcile the oneness of Indian Philosophy with the manyness of Science, I found light in a sentence of Bergson that "quantity is incipient quality." Conversely, we may say that quality is a range of quantity. A range of frequencies yields us the quality of colour. Another range of frequencies yields us the quality of sound, Perhaps with greater knowledge, we could find the ranges of number which yield us the qualities associated with the other senses. Mathematics is not merely the texture of the thought of man but also of the being of matter. Nature rarely makes mistakes of arithmetic in her creation. Cotton may be defined by the chromosome number 13. Asia and America may be defined by the multiples of 13 that cotton presents when developing in these continents. The succession of chromosome numbers at different levels seems to yield the hierarchy of life—plants and animals, mosquitoes and men. Agro-biology yields equations valid for many crops, in many countries, under widely varying conditions. Mathematics is thus not merely an emanation from the mind. Equally it is an emanation from matter. May not mathematics be then also an emanation from spirit from which both mind and matter emerge? In India, mathematics like other forms of thought, has flowed from spiritual intuition. In the school of Relativists to which Eddington belongs, we see today a reverse flow from mathematics to mysticism.

The phrase, mixed mathematics, may thus take a wider connotation than it has hitherto done. Mathematics may be enriched not merely by the mixture of mind and matter but also by the mixture of mind and spirit. It may not only construct a bridge into the outer world from mind to matter but also a bridge into the inner world from mind into spirit. Ranges of quantity yield quality. The summation of all quality is the One, that is the supreme class of classes. Conversely, the unity of the Universe is the die that marks off the units of the Universe.

At the request of Dr. Narasinga Rao, two quotations from my recent speech at the Convocation of the Andhra University are included in this Exposition. The first runs thus :—

"Mathematics is the lily from the pond of life,
Rising straight from the depth of life,
Floating easily on the surface of life, and
Converting the inscrutable mud of the pond into
simple beauty of form and colour.

This is illustrated by a painting where Baskaracharya sits under the Peepul tree on the bank of the village pond and shows to Leelavathi the beauty of the Ganita Pankaja, as it rises from the mud of the pond, grows through the water, seeks the Sun, waves in the air and weaves the beauty of space and ether in form and colour. As Bhaskaracharya, philosopher as well as Mathematician, lists the pancha bhutas, the lotus too traces them in that order, in that fulness. The rhythm of the mind of man is answered by the rhythm of the body of the lotus. This brings us to the second quotation :—

"The pattern of life is set to two different rhythms,
The rhythm of nature and the rhythm of man,
The rhythm of matter and the rhythm of mind.
Nature makes the woof,
Man makes the weft,
Time, the rhythm of life, is two dimensional."

The first quotation has been depicted in colours. Let me give you a word picture of how the second quotation may be so depicted.

In the left of the painting is the Earth revolving round its axis with clouds of life flowing from it spirally. From this flow of life, horizontal lines of time are spun by the many fingers of Brahma, shown at the extreme right. Across these lines, the life of men rises and falls, forming a graph not of points but of pictures. At the left, there are four pictures showing life flowing at a low pitch—path-finder, weaver, builder and cultivator. In the middle the graph rises and falls. At the right it rises to a high crescendo, showing four pictures—Scientist, Artist, Philosopher and Saint. The fingers of the Saint are linked with the fingers of Brahma as height merges into length.

Mathematics has thus admittance to a wide range of life. It is, like us, a Manasaputra. We in India have thought many thoughts, seen many visions, built and destroyed many edifices. The mind of India, like the body of India, has been weathered by fine breezes, passionate cyclones and richly laden monsoons. All round India is the dust from her pores. And let me say to you, Indian Mathematicians, build from the dust of the Indian mind, the dust that comes of disintegration in order to form the vehicle of a new construction, so it may be said that India is on a holy quest for the lotus of mathematics, the Ganita Pankaja.

I have now much pleasure in declaring this Exposition open.

ABSTRACTS OF PAPERS

Dr. A. N. Singh, Lucknow University.

The South Indian Schools of Mathematics in the early and middle ages.

R. Krishnamurti, Nizam's College, Hyderabad.

Precession in Hindu Astronomy.

Whitney's statements regarding the knowledge of precession in ancient Hindu astronomy are questioned and his criticisms shown to be based on a mistranslation. It is pointed out that there are references to precession in the Vedas (vide *Taitreya Aranyaka*, first *Prasna*, fifth *Anuvak* where the two halves of the ecliptic are described as well strung bows and where it is stated that Indra in the form of a white ant cuts away the bow string, that the portion cut off is added to the head, and that this is the cause of Pravargya and so on; and the next *Anuvak* where the period is given as 28,000 years). It is shown that the concept of precession has been the same all along i.e. a libration of Aswini on the horizon of the observer and the revolution of the moving vernal equinox along the ecliptic.

Appaji Josier, Coimbatore

Vakyam and Drik Ganitham in Hindu Astronomy

Mukunda Marar and C. T. Rajagopalan,
Christian College, Tambaram.

On the Hindu Quadrature of the Circle.

The material of this paper is drawn mainly from *Yukti-Bhāṣa*, a Malayaḷam work in manuscript, of uncertain date and authorship.

The first section of the paper gives the geometrical proof, substantially as it stands in the *Yukti-Bhāṣa*, of Gregory's series for the inverse tangent. The second section discusses the method explained in *Yukti-Bhāṣa* for converting Gregory's series for $\pi/4$ into other alternating series whose nth terms are $O(1/n^{2p+1})$ as $n \to \infty$, $p = 1, 2, 3, \ldots$ This method gives us at the same time an

endless sequence of approximations to $\pi/4$ in which every approximation is better than the preceding. For $p=2$ the series is

$$\frac{\pi}{16}=\frac{1}{1^5+4\cdot1}-\frac{1}{3^5+4\cdot3}+\frac{1}{5^5+4\cdot5}-.$$

and the approximation:

$$\frac{\pi}{4}\doteqdot 1-\frac{1}{3}+\frac{1}{5}-\dots\pm\frac{1}{n}\mp\frac{(n+1)/2}{(n+1)^2+4}$$

By treating the series

$$\frac{\pi-2}{4}=\frac{1}{2^2-1}-\frac{1}{4^2-1}+\frac{1}{6^2-1}-\dots,$$

in the same way as Gregory's series for $\pi/4$, we obtain a second sequence of approximations and associated alternating series whose n-th terms are $O\ (1/n^{2p+2})$, $p=1, 2, 3, \dots$

The concluding section discusses the likelihood of these Hindu investigations in circle-squaring having appeared first in the 15th century, as a persistent Kerala tradition would have us believe. (*vide* quotation given by Professor A. A. Krishnaswami Ayyangar under Question 1775, *Mathematics Student,* Vol. VII 1939, p. 79) The tradition, even if it has not been accepted finally, casts grave doubt on G. R. Kaye's statement that 'after the time of Bhāskara (born A. D. 1114) no Indian mathematical work of historical value or interest is known.'

Nazir Ahmad Chaudhary, Aligarh

The Contribution of Muslims towards Mathematics

The paper gives a brief survey of Mathematics under Arab influence. The works of Al-Bairuni the greatest Arab Mathematician are dealt with in detail, especially his theory of Natural Numbers.

K. Nagabhushanam and **K. Sambasiva Rao,** Andhra University.

Vinogradow Numbers II.

The work of this paper is a continuation of our earlier paper entitled "A note on Vinogradow Numbers" (*Proc. Ind. Ac. Sci.* Vol. 17). Some necessary conditions and sufficient conditions for a number to be a non V. N. have been obtained. It is shown that non-Vinogradow Numbers are nondenumerable and everywhere dense while they are of measure zero. The existence of non-Vinogradow Numbers was first pointed out to us by Dr. Vijayaraghavan.

K. Sambasiva Rao, Andhra University.

On the distribution of a class of primes.

Let $a_1, a_2, \ldots a_m$ be given integers. The asymptotic distribution of primes p for which $p+a_1, p+a_2, \ldots p+a_m$ are all primes is an unsolved problem which Hardy and Littlewood considered. A particular case of the above, the asymptotic distribution of primes p, $p+2$ is a wellknown unsolved problem. Taking the analogy from the above S. S. Pillai worked out the distribution of square-free integers q for which $q+a_1, \ldots q+a_m$ are all square free. In this paper the asymptotic distribution of primes p for which $p+a_1, \ldots p+a_m$ are all square free is considered. If $\pi\ (N;\ a_1, \ldots a_m)$ denotes the number of such primes less than N, then

$$\pi\ (N;\ a_1, \ldots a_m) = \text{Li}\ NA + O\{N/(\log N)^h\}$$

where h is a constant >1, $\text{Li}\ N = \int_2^N \frac{dx}{\log X}$ and

$A = \prod_p \left(1 - \frac{1}{p(p-1)}\right)$, p running through all primes not dividing $a_1, a_2, \ldots a_m$

Also, a generalisation of a certain class of additive problems considered by T. Estermann, A. Page, Walfisz, Erdos and Pillai has been obtained in this paper. This result includes the results of all the above writers.

S. S. Pillai, Calcutta University

(1) *On the Maximum order of $d(n)$*

With the help of Highly Composite Numbers, Ramanujan determines the maximum order of $d(n)$. The same is done here directly from the consideration of $(x+1)/p^{\alpha x}$.

(2) *On Waring's problem with powers of primes*

Let $G_1(k)$ denote the least value of s such that every *large* number is the sum of s, kth powers of primes. p^θ is the highest power of a prime p which divides k. $\gamma = \theta + 2$ for $p = 2$, $2|k$, and $\gamma = \theta + 1$ otherwise. $K = p^{\gamma_1} \ldots p_n^{\gamma_n}$, where $p_1, \ldots, p_n$ are primes for which $(p_i - 1)|k$, $i = 1, 2, \ldots n$. $t(k)$ is the least value of s such that the congruence

$$N \equiv x_1 + x_2 + \cdots + x_s \pmod{K}$$

is solvable for every N, where $x = 0$ or the kth power of a prime. Then, it is proved that

(1) $G_1(k) \geqslant 1 + k + t(k)$.

(2) $t(k) \leqslant p_1^{\gamma_1} + p_2^{\gamma_2} + \cdots + p_n^{\gamma_n} - n + 1$.

(3) *On Bertrand's Postulate*

Cebicev was the first to prove Bertrand's postulate. Ramanujan simplified the proof. Here the proof is further simplified. In this proof Stirling's approximations are not used. It is based on Landau's start namely, $\frac{(2n)!}{m!\,n!}$.

S. Chowla, Government College, Lahore.

The biquadratic residues (mod 101) and (mod 197).

Let p denote a prime of the form $4r+1$. What primes p have the following property? Let $r_1, r_2, \ldots, r_m$ where $m=(p-1)/4$ denote the biquadratic residues (mod p). The number of solutions of $n \equiv r_i - r_j$ (mod p) where $1 < n < p$ and i, j take all different values up to m, *has the same number of solutions for every n.* It is proved in this paper that $p=37$, $p=101$, $p=197$ are solutions of the problem. Of these the first case namely $p=37$ is probably known. There are applications to problems on "complete l-m-n configurations" (vide. Carmichael's *Group Theory*). A large number of special cases have been solved by Mr. R. C. Bose, who has made a very penetrating study of the subject.

L. G. Sathe, Calcutta.

On a congruence property of the divisor function.

Let $d(n)$ be the number of divisors of n and $N(k, x)$ the number of $n \leqslant x$ for which $d(n) \equiv 0 \pmod{k}$.

I show that

THEOREM I:—$N(k, x) = cx + O\left(x^{1/(m_1-1)+\varepsilon}\right)$ for every $\varepsilon > 0$ where $c = c(k) > 0$ is a constant and $k = m_1 m_2$, $m_1 < m_2$; m_1 and m_2 being different odd primes.

THEOREM II:—$N(2k, x) \sim N(k, x)$ when k is odd.

THEOREM III:—$N(k, x) \neq O(x)$ for any k.

Let $d(n)$ be the number of divisors of n and $N(3, r, x)$ the number of $n \leqslant x$ for which $d(n) \equiv r \pmod 3$. It is shown that when $r=1$ or 2,

$$N(3, r, x) = \frac{3}{\pi^2}\,\zeta(3).\, x + O\left(\frac{x}{(\log x)^t}\right) \text{ for every } t;$$

and

$$N(3,\ r,\ x) = \frac{3}{\pi^2} \zeta(3).\ x + O\left(x^{0\cdot 83}\right)$$

on assuming Riemann-hypothesis concerning the roots of the ζ - function.

K. G. Ramanathan, Madras University

Congruence properties of $\sigma(n)$ the sum of the divisors of n.

It is proved that

$$\sigma(3n-1) \equiv 0 \pmod{3}$$
$$\sigma(4n-1) \equiv 0 \pmod{4}$$
$$\sigma(6n-1) \equiv 0 \pmod{6}$$
$$\sigma(8n-1) \equiv 0 \pmod{8}$$

These imply in turn

$$\sigma(12n-1) \equiv 0 \pmod{12}$$
$$\sigma(24n-1) \equiv 0 \pmod{24}.$$

It appears that there are no such simple congruences to other moduli.

Hansraj Gupta, Government College, Hoshiarpur

Congruence Properties of $\tau(n)$

Defining $\tau(n)$ as the coefficient of x^n in the expansion of

$$x\{(1-x)(1-x^2)(1-x^3)\ .\ .\ .\}^{24}$$

Ramanujan stated, and Mordell proved that

$$\tau(mn) = \tau(m)\cdot\tau(n), \text{ provided } (m, n) = 1.$$

Some congruence properties of $\tau(n)$ with regard to the moduli 5, 7 and 23 were also proved by Ramanujan. In this paper, it is proved that

$\tau(n) \equiv 1 \pmod{2}$, if and only if n is an odd square;

$\tau(p) \equiv 2 \pmod{4}$, if p be a prime of the form $4k+1$, $k > 0$;

$\equiv 4 \pmod{8}$, if p be a prime of the form $8k+3$; $k \geqslant 0$;

while

$\tau(8n+7) \equiv 0 \pmod{8}$, $n \geqslant 0$.

D. R. Kaprekar, Devlali

Orthogonal demlofication of numbers

A Demlo number is one of the form M $(R)_n P$ where $(R)_n$ stands for the digit R repeated n times, and M and P are numbers whose sum is of the form $(R)_m$. Such a Demlo number has necessarily a factor of the form $F = 1 + 10 + 10^2 + 10^3 + \cdots$ If the semi demlo number $(R)_n P$ is demlofied, *i.e.*, multiplied by a number of the form F, we get certain types of numbers whose properties are discussed.

M. V. Subba Rao, Madras University.

On Ramanujan's Trigonometrical Sum $c_m(n)$.

This function of Ramanujan, $c_m(n)$ defined as the sum of the nth powers of the primitive m^{th} roots of unity, is here shown by direct methods, to satisfy the relations:

(i) $c_m(n) = \mu\,[m/(m, n)] \cdot \phi(m) \div \phi[m/(m, n)]$.

(ii) $c_m(1) - \frac{1}{2} c_m(2) + \frac{1}{3} c_m(3) - \cdots = \Lambda\left(\frac{m}{2}\right)$

(iii) $c_m(n) = f(m, n)$, Von Sternuck's function, which is the excess of the number of partitions of n into distinct parts mod m, the number of parts being even, over the same, the number of parts being odd.

The method of proving (iii) is used to find the total number of partitions; and also to deduce similar results. For example: "The number of partitions of n into distinct parts mod m, (m odd), the parts being all numbers less than and prime to m, does not depend upon n, and is $[2^{\phi(m)} - 1]/m$; and the total number of parts in all these partitions is

$\phi(m)[2^{\phi(m)} - 1]/2m$ if $m \nmid n$; and $[2^{\phi(m)} + m - 1]\ \phi(m)/2m$,

if $m|n$. Thus if $m=5$, $n=3$; we have $3 \equiv 1+2 \equiv 4+3+1 \equiv 3 \mod 5$ so the number of partitions is 3, and total number of parts is 6.

Abdul Majid Mian and S. Chowla, Lahore

The differential equations satisfied by certain functions

Let $\sigma(n)$ denote the sum of the divisors of n and $p(n)$ the number of partitions of n. We prove that the power series

$$\sum_{1}^{\infty} \sigma(n)\, x^n$$

and

$$\sum_{1}^{\infty} p(n)\, x^n$$

where $|x|<1$, satisfy algebraic differential equations. We believe, but cannot prove that $\sum_{1}^{\infty} d(n)\, x^n$ where $d(n)$ is the number of divisors of n *does not* satisfy an algebraic differential equation.

K. Nagabushanam, Andhra University

A Property of Algebraic Numbers

If θ is any real algebraic irrational number, and $h>1$, a constant it is here shown that there exists a constant $c=c(h)$ such that

$$\left|\sum_{p\leqslant N} e^{2\pi i\theta p}\right| < \frac{cN}{(\log N)^h},$$

where p runs through primes, and N is any positive number. These sums are of interest in Diophantine approximations.

M. Venkatarama Aiyar and M. Bhimasena Rao, Bangalore

On Types of Solutions of $x^3+y^3+z^3=1$ in integers

A paper on this subject has been printed in the J. I. M. S., and others have been read at previous meetings. In this paper Type 19 is fully worked out and some general results common to several types are derived.

A. E. Palkar, Kolhapur

On the Equation $n!+1=m^2$

There are three known solutions of this equation corresponding to $n=4$, 5, and 7 and it has been conjectured that there are no other solutions. With the help of the assumption that if N is a large composite number of the form $p_1^a\, p_2^b\, p_3^c \dots p_n^k$ where $p_1, p_2, p_3 \dots$ are small primes as compared with N arranged in increasing order and the majority of the indices $a \geqslant b \geqslant c \geqslant \cdots$ with $p_n < N^{1/c}$, then N can be always resolved into one pair of composite factors which are equal or almost equal, it is shown that the number of solutions is finite.

Further, with the help of the extended assumption it is shown that $n!+l^a=m^a$ has a finite number of solutions.

Amrit Sagar Puri, Kapurthala

On some special classes of simultaneous equations

The paper discusses a method of finding the number of real distinct pairs of roots of a special class of simultaneous equations $f_1(x, v)=0$, $f_2(x, v)=0$. These classes are specified by the fact that f_1 and f_2 satisfy one of the two equations

$$f(x+\omega y)=f_1(x, v)+\omega y\, f_2(x, v)$$
$$f(x+iy)=f_1(x, v)+iy\, f_2(x, v)$$

where $f(z)$ is a polynomial and $\omega^2=1$, $i^2=-1$ while $y^2=v$.

S. Chowla, Government College, Lahore

On a certain system of equations considered by Ramanujan

The system in question is

$$x^2=a+y$$
$$y^2=a+z$$
$$z^2=a+x$$

Elimination of y, z leads to an equation of degree 8 in x, solved by Ramanujan.

If a is a rational number such that $4a-7$ is the square of a rational number (e.g., $a=\frac{37}{16}$) it seems (what I have not proved) that all the roots are rational functions of roots of unity. When $a=\frac{37}{16}$ I found that the equation of degree 8 in x has 2 roots which are expressible as rational functions of 41st roots of 1; 3 roots are expressible as rational functions of 31st roots of unity. I communicated these results to Prof. S. N. Bose (Dacca) who expressed the remaining 3 roots as rational functions of 43rd roots of unity.

The general problem, is: given a such that $4a-7$ is a perfect square; find a general method for expressing the 8 roots as rational functions of roots of unity.

T. Venkatarayudu, Hindu College, Masulipatam.

The characters of the symmetric group of degree 14.

Littlewood[1] calculated the tables of characters of the symmetric groups of degrees 1 to 10. Later M. Zia-Ud-Din[2] calculated the same for symmetric groups of degrees 11, 12 and 13. Here we

1. The theory of group characters and Matrix representations of Groups. 1940.
2. *Proceedings of the London Mathematical Society* 1935, 39, 200
   Do. 1937, 42, 340.

obtain the character table for the symmetric group of degree 14. The number of partitions of 14 is 135 and therefore there are 135 irreducible representations of the symmetric group and 135 conjugate classes of elements. The three partitions $[721^5]$, $[6321^3]$, $[542^21]$ are self conjugate and the remaining 132 partitions can be divided into 66 pairs of conjugate partitions. In the following table the characters corresponding to one from each pair of conjugate partitions and those corresponding to the self conjugate partitions only are given. The characters corresponding to the remaining 66 partitions can be obtained by using the character relation,

$$\chi_\rho^{p} = \chi_\rho^{p'} \chi_\rho^{[n]} \qquad p \text{ is conjugate to } p'$$

The characters of the class (14) are obtained by using the formula derived by the author[1] and the remaining table is obtained with the help of the recurrence relations between the S-functions. The table is finally checked with the help of the orthogonality relations between the group characters and the formulae derived by the author for the characters of the class (n_1, n_2) in the symmetric group of degree n_1+n_2. The order of a class (that is the number of elements in a conjugate class of elements of the symmetric group) $1^a\ 2^b\ 3^c \ldots$ is

$$\frac{n!}{a!\ b!\ c! \ldots 1^a\ 2^b\ 3^c \ldots}$$

K. Chandrasekharan, Andhra University.

Partially ordered Sets and Symbolic Logic

In this paper, a concise account is given of the various algebras of logic by the introduction of lattice-theoretic ideas. The System of Material Implication due to Russell and Whitehead, the System of Strict Implication due to Hewis, Heyting's formalization of Intuitionistic Logic and the many-valued logics of Lukasiewicz and Tarski are dealt with. The concept of a Truth-value System and its possible use are explained.

M. Raja Rao, Bangalore.

Irrational Power as a section.

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1. The characters of the class (n_1, n_2) of the symmetric group of degree n_1+n_2. *Journal of the Indian Mathematical Society.* (Under publication.)

Rev. Fr. C. Racine, Loyola College, Madras.

On Green's Theorem.

Green's theorem, expressed by the equation

$$\iint \Delta U d\sigma + \int \frac{dU}{dn} ds = 0$$

is proved under the following conditions:

(1) U and its first derivatives are continuous in the closure of a domain D;

if $k = ah$, a being fixed, and if we put $\Delta(u, h, x, y) =$

$$\left\{ \frac{U(x+h, y) + U(x-h, y) - 2U(x, y)}{h^2} + \frac{U(x, y+k) + U(x, y-k) - 2U(x, y)}{k^2} \right\}$$

then $\lim\limits_{h \to 0} \Delta(U, h, x, y)$ exists at every point of the domain D is and integrable over D;

$\Delta(U'_x, h, x, y)$ and $\Delta(U'_y, h, x, y)$ are uniformly bounded in D;

(2) the boundary C of D is a limit of polygons entirely contained in D;

(3) on that curve, the integral $\int \frac{dU}{dn} ds$ has a meaning.

This result is a generalisation of a paper read at the Indian Science Congress, Baroda, in 1942.

S. M. Shah, Muslim University, Aligarh.

Note on entire series with positive coefficients.

Let $f(z) = \sum\limits_0^\infty a_n z^n$ be an integral function of order ρ and let $a_n \geqslant 0$. It is known that (Polya und Szego *Aufgaben and Lehrsatze* Band 2 Ex. 53 P 8)

$$\limsup_{r \to \infty} \frac{\log \{rf'(r)/f(r)\}}{\log r} = \rho$$

I prove that

$$\liminf_{r \to \infty} \frac{\log \{rf'(r)/f(r)\}}{\log r} = \lambda$$

where λ is the lower order of $f(z)$.

Without imposing any condition on a_n, I prove

$$\limsup_{r\to\infty} \frac{\log \{r M_1(r)/M(r)\}}{\log r} \leqslant \frac{3\rho}{2}.$$

V. Ganapathy Iyer, Annamalai University

On singular functions

1. A function $f(x)$ possessing derivatives of all orders in (a, b) is said to be singular in (a, b) if (i) $f(x) \neq 0$ in (a, b) and (ii) there is a point x_0 in (a, b) such that $f^n(x_0) = 0$ for $n = 0, 1, 2, \ldots$.

The function is said to be singular at a point x_0 if (ii) holds and if $f(x) \neq 0$ in any neighbourhood of x_0.

The existence of singular functions is known and they are responsible for the failure of the uniqueness theorem for Taylor Series. Some properties of singular functions and their relation to quasi-analytic classes are discussed. In particular, I prove the following two results :

(i) if $f^n(x) \geqslant 0$ for all n and for all x in (a, b), then $f(x)$ is analytic in $(a, b - \delta)$, $\delta > 0$, and cannot have a singular point in (a, b);

(ii) if each $f^n(x)$ keeps the same sign in (a, b) [though this may differ for different n], then $f(x)$ cannot have a singular point in (a, b).

S. Minakshisundaram, Andhra University

A Note on the theory of almost periodic functions

The problem of summability by typical means of the Fourier series of an almost periodic function is studied, for the purpose of which 'Bessel Summability' is made use of.

We may state the following theorem as an instance. Given an even uniformly almost periodic function $f(x)$, with,

$$f(x) \sim \sum_{0}^{\infty} a_n \cos \lambda_n x$$

there exists an index $\alpha \geqslant 0$, so that the series on the right is summable (R, λ, k) for $k > \alpha$ to $f(x)$ at every point x.

Hari Shanker, Anglo Arabic College, Delhi.

(1) *Some Integral Representations for Whittaker functions*

In this paper some integral representations for Whittaker functions have been obtained by the method of Operational Calculus, *e. g.*

(i) $W_{k,m}(x)$

$$= \frac{\Gamma(\frac{1}{2}-\lambda+\mu)\, x^{m-\mu}}{\Gamma(\frac{1}{2}-k+m)\,\Gamma(2\mu-2m)} \times$$

$$\int_0^\infty e^{-t/2}\, t^{2\mu-2m-1}(1+t/x)^{-\mu-\frac{1}{2}}\, W_{\lambda,\mu}(x+t) \times {}_1F_1[k-\lambda+\mu-m;\, 2\mu-2m;\, t]\, dt$$

provided $R(\frac{1}{2}-\lambda+\mu)>0$, $R(\frac{1}{2}-k+m)>0$, $R(\mu-m)>0$.

(ii) $W_{k,m}(\frac{1}{2}x^2)\, e^{-\frac{1}{4}x^2}\, x^{2l-\nu-2}$

$$= \frac{\Gamma(l-m+\frac{1}{2})\,\Gamma(\frac{1}{2}+l+m)}{\Gamma(l-k+1)\,\Gamma(\nu+1)}\, 2^{l-\nu-1}$$

$$\int_0^\infty t^{\nu+1}\, J_\nu(xt)\, {}_2F_2\left[\begin{matrix} l+m+\frac{1}{2},\ l-m+\frac{1}{2}; \\ \nu+1,\ l-k+1;\ -t^2/2\end{matrix}\right] dt.$$

provided that the integral converges.

(2) *On the expansion of Whittaker function in series of Whittaker functions.*

In this paper the following expansions have been obtained:—

(i) $W_{k+n/2,\, m+n/2}(x)$

$$= \frac{x^{n/2}}{\Gamma(n)\,\Gamma(1/2-k+m)} \sum_{r=0}^{\infty} \frac{\Gamma(n+r)\,\Gamma(m-k+r+1/2)}{r!}\, W_{k-r,\, m}(x)$$

where $R(1/2-k\pm m)>0$.

(ii) $W_{k+n,\, m}(x)$

$$= \frac{\Gamma(1/2-k-m)}{\Gamma(n)\,\Gamma(1/2-k-m-n)} \sum_{r=0}^{\infty} \frac{\Gamma(n+r)}{r!}\, x^{r/2}\, W_{k-r/2,\, m+r/2}(x)$$

where $R(\frac{1}{2}-k\pm m)>0$, $R(\frac{1}{2}-k\pm m-n)>0$.

A. A. Krishnasawmi Ayyangar

(1) *Some Topological notes on the point and Line elements in Euclidean Geometry*

There have been several discussions recently in the Mathematical Gazette about the subject of this paper and all of them have missed the important role of the cross-ratio in determining the topological

property of 'separation'. An important theorem in this connection is:— The point (x_1, y_1) lies in the obtuse (or acute) angle between the lines

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

if $$(ax_1^2 + 2hx_1 y_1 + by_1^2 + 2gx_1 + 2fy_1 + c)\ (a + b)$$

is positive (or negative), given $a + b \neq 0$.

(2) *A cardioide associated with two intersecting circles*:

If two circles intersect at A and B and any straight line through A cuts the circles at P and Q, the locus of the intersection of tangents at P and Q is a cardioide with cusp at B, touching the two given circles at A_1 and A_2 where AA_1, AA_2 are tangents at A to the circles. The diameter of the cardioide is also a diameter of the circle through B and the centres of the circles.

This problem is discussed in the paper with some generalisation and history of its genesis.

K. Rangaswami Ayyar, Annamalai University

The Theory of the Durairajan point of a Quadrangle

Given four points A, B, C, D in a plane, the six circles of similitude of the four circles BCD, CAD, ABD, ABC taken in pairs all meet in a point M. This point has been called the Durairajan point of the quadrangle ABCD by Mr. V. Ramaswamy Ayyar and has been the subject of study by him, Mr. A. A. Krishnaswamy Ayyangar and a number of others. In this paper both metrical methods and those of circle geometry are used, by considering M as the isogonal conjugate of the unique point at infinity in the generalised isogonal transformation defined by the tetrad of point circles A, B, C, D. The results obtained may serve to present in a connected form the solutions to many of the problems proposed by Mr. Ramaswamy Ayyar pertaining to the Durairajan point, including the one relating to "an intriguing fallacy."

A. Narasinga Rao and **M. Venkataraman**, Annamalai University

On Hart-System of spheres in hyper-space

Two sets of four circles each, of which every member of one set touches every member of the other set, constitute a plane Hart

system. It is known that there are two such types, one—the flat variety, in which the circles of each tetrad have a common orthogonal circle, and the general variety in which this condition is not fulfilled. This paper discusses the analogues of the flat variety in hyper space.

V. Ramaswami, Andhra University

On Dr. Narasinga Rao's problem of the tetrahedron

M. Venkataraman, Annamalai University

(1) *On a chain of theorems on rectangular hyperbolas*

Two rectangular hyperbolas determine a circle—the centre locus of the pencil containing them. Three rectangular hyperbolas give 3 such circles which are proved to pass through a point. Given 4 rectangular hyperbolas whose centres are concyclic, the 4 points determined as above (from sets of 3 of these) all lie on a circle. Given 5 rectangular hyperbolas whose centres are concyclic, the 5 circles so obtained are concurrent. These incidence relations are shown to continue indefinitely.

(2) *On a chain of theorems in circle-Geometry*

Mr. B. R. Venkataram has established that the 4 radical centres of the sets of three of 4 circles with concyclic centres lie on a cirlcle, that given 5 circles with concyclic centres, the 5 circles obtained as above from sets of 4 of these are concurrent; and that the incidences continue up to 7 circles.

It is proved here that the incidences continue indefinitely and that like results do not hold in higher dimensions.

(3) *On the deformation of a conicoid*

Prof. Cayley has proved that when a conicoid is deformed so that the generating lines are subjected to rigid movements we obtain a confocal system of quadrics.

A like characterisation is obtained in the case of a quadric whose circular sections are subject to rigid motion during the deformation.

B. Seetharama Sastry, Central College, Bangalore.

Crevices in Pyramids of Spheres.

The different types of crevices that form the free space inside a regular tetrahedral pyramid formed out of equal spheres, and also the corresponding crevices for a pyramid on a square base are studied in this paper.

Ram Behari, University of Delhi.

A Note on Rectilinear Congruences.

Properties of the ruled surfaces through a line of a rectilinear congruence are obtained which will ensure that the congruence is normal.

V. Seetharaman, Annamalai University.

On the existence of a metric for higher path-spaces.

In this note is considered the conditions under which the path equations $x^{(\sigma+1)i} + \Gamma^i\ (xx' \ldots x^{\sigma},\ t) = 0$ are identifiable with the extremals $\delta \int f dt = 0$. We, prove the result that the necessary and *sufficient* condition under which the problem is solvable is that the equations of variation of the path equations be self-adjoint. An analysis of the self-adjoincy conditions reveals also the particular form of Γ^i. The discussion slightly differs for the two cases of $(\sigma+1)$ even or odd.

V. S. Krishnan, M.A., Madras University

On the equivalence of the generalisations of metric space by Andre Weil and G. Baley Price

The object of the paper is to establish the identity of two topologisations of an abstract space given by André Weil and G. Baley Price respectively. Thus I show that Weil's "space of uniform structure" is also "a generalisation of a metric space" as defined by Price; and, conversely, Price's "generalisation of a metric space" becomes a 'space of uniform structure on the assumption of one extra hypothesis (which is given by Price himself as Hypothesis III.)

S. R. Das, Commercial College, Delhi

Determination of the simplest possible scheme of equivalent planes in aeroplane motion

Captain Ferber discusses this question by assuming that the surfaces of an aeroplane can be replaced by three equivalent fictive

surfaces in three planes mutually at right angles. This is true in certain cases, but not where he assumes that a surface can be replaced by its projections and that the error will be small. This paper attempts to discuss the cases when Feber's equations break down.

B. R. Seth, Delhi.

On Guest's law of elastic failure.

Various hypotheses have been advanced for determining the yield point of an elastic material. In one group we may combine the hypotheses of (1) principal maximum stress (2) maximum principal strain (3) maximum shear stress or maximum principal stress difference. Of these the last one is favoured by many writers.

A second group of theories starts from the idea that there is a limit to the amount of strain energy which can be stored in the element of volume of an elastic solid.

Very few have found their way to agree to Guest's criterion that besides the shearing stress, which is a primary factor in yield, volumetric stress of a certain type should also be taken in account. The theory of the finite strain developed in some recent papers gives a criterion similar to that of Guest. If p_1, p_2, p_3, are the principal stresses, Guest uses $(p_1+p_2)/2$ as the correction. But, if the correction is to reflect the change in volume, the average volumetric stress $(p_1+p_2+p_3)/3$ seems to be more feasible. This is the result we obtain from the theory of finite strain.

A. A. Krishnaswami Ayyangar, Maharaja's College, Mysore.

(1) *Random numbers*

Random numbers are a set of numbers which taken as a whole do not show bias of any kind. Any mathematical property of integers may be used to test this bias. There are several ways of generating random sets. Tippett, Fisher, Kendall and Mahalanobis have published different sets of random numbers. I have attempted to collect a set of five-digit numbers from students who were asked to write down without any thought or design two rows of five digits each, on pieces of paper supplied to them for the purpose. This collection of numbers was then submitted to a 'randomising sieve' which consists of a set of tests such as frequency test, block test and gap test and I have recorded the survival of the fittest. This new set of random numbers is expected to be as satisfactory as any of its predecessors.

(2) *An Investigation concerning public examination results*

In the annals of public examinations it is almost axiomatic that 80 % of passes with 40 % as average mark should be deemed a perfect result and examiners very often strain to reach this ideal even when it is sometimes mathematically impossible. The two variables, (x) the percentage of passes and (y) the average mark, cannot be arbitrarily fixed beforehand in defiance of an underlying relation which it is our aim to discover. That such a relation exists is obvious from a consideration of the extreme cases where (y) is 0 and 100. It is therefore reasonable to expect that for every (x) or (y), there are certain specific limits within which (y) or (x) should lie and the violation of this principle must imply arbitrary tampering with the results. Generally, as examination results are more or less moderated to fall in with a predestined set of values (x, y) calculated to placate public criticism, the examination results are generally untouchable for statistical scrutiny and one has to fall back on empirical methods of discovering the relation between (x) and (y). The present paper employs Lorenz Curve for this purpose and obtains a quadratic relation of the form

$$x^2(a+by)+x(c+dy)+e=0,$$

where a, b, c, d, e are appropriate constants based upon the form of Lorenz Curve and the minimum mark for a pass.

M. V. Jambunathan, Mysore University

(1) *Measures of kurtosis of some platykurtic distributions ;*

(2) *Upper and lower limits of the Pearsonian Ratio*

Pearson's trisection rule, namely

$$(\text{Mean}-\text{Mode})=3\ (\text{Mean}-\text{Median})$$

does not always hold, as has been pointed out by J. B. S. Haldane (*Biometrika*, Vol. XXXII, Parts III and IV, April 1942).

Taking the single case of the scalene triangle distribution, Haldane has shown that the median is one quarter and not one third of the distance from the mean to the mode, if $(a-b)$ is small compared with a.

What happens when this provisio is relaxed is not investigated by Haldane, and the above statement regarding the approximate value of the ratio is misleading. Mr. A. A. Krishnaswamy Ayyengar

has shown, employing geometrical methods, that the ratio may take any value from 4 to $4+3\sqrt{2}$. An analytical proof of this result which fixes the upper and lower limits to the Pearsonian ratio is given in this paper.

Afzal Ahmed, Osmania University

On the theory of linear equations in the quaternionic field

The author has attempted to prove that the system of infinite linear equations in an infinite number of unknowns

$$\sum_{k=1}^{\infty} a_{ik}\, x_k = y_i \qquad (i=1,\ 2,\ \cdots)$$

is soluble in H. Wachs 'Generalized Hilbert space' under the following conditions which are both necessary and sufficient:

If $A=(a_{ij})$ is an infinite matrix then

(1) $\overset{+}{A}A$ is a matrix of bounded form.

(2) $(\overset{+}{A}Ax,\ x)$ is positive definite.

where $x=(x_1,\ x_2,\ \ldots)$ is a vector in the generalized Hilbert space.

(3) The operators $[\overset{+}{A}A]_n$ do not have their eigen value zero as a limit point.

GLEANING

Some time ago my late Papa
Acquired a spiral nebula.
He bought it with a guarantee
Of content and stability.
What was his undisguised chagrin
To find his purchase on the spin.
Receding from his call or beck
At several million miles per sec ,
And not, according to his friends,
A likely source of dividends.
Justly incensed at such a tort
He hauled the vendor into court,
Taking his stand on Section 3
Of Bailey " Sale of Nebulae."
Contra was cited Volume 4
Of Eggleston's " Galactic Law "
That most instructive little tome
That lies uncut in every home.
" Cease " said the sage " your quarrel base ;
Lift up your eyes to Outer Space.
See where the nebulae like buns
Encurranted with infant suns,
Shimmer in incandescent spray
Millions of miles and years away.
Think that, provided you will wait,
Your nebula is Real Estate,
Sure to provide you wealth and bliss
Beyond the dreams of avarice.
Watch as the rolling æons pass
New worlds emerging from the gas :
Watch as the brightness slowly clots
To eligible building lots.
What matters a depleted purse
To owners of a Universe? "
My father lost the case and died :
I watch my nebula with pride
But yearly with decreasing hope
I buy a larger telescope.

(*From a Chinese Astronomer*)

POPULAR LECTURE ON

RANDOM LINES AND DIGITS[1]

BY

A. A. KRISHNASWAMY AYYANGAR. *Maharaja's College, Mysore*

The word 'random' according to the dictionary, signifies "without any purpose or design". But this is rather vague. How vague it is will become evident if I ask you to draw three random lines and write down two rows of five digits at random. For, however much you may try to avoid all design, I can prove to you by statistical methods that you will not really succeed in doing so. In your trilinear signatures an unconscious flow of your individual character and personality can always be perceived. Probably, no man would like to wear his heart on his sleeve for statistical daws to peck at! But the very desire to suppress one's personality may let loose the unconscious aspects of it. In the Bhagavad-Gita. Chap. 3, verse v, we read

कार्यते ह्यवशः कर्म सर्वः प्रकृतिजैर्गुणैः ।

'Everyone is compelled to act in consonance with the qualities of his nature'. It is perhaps impossible to abandon oneself so completely and bottle up one's personality so effectively as not to let out even a small streak of it to show itself. Possibly a Yogi may be able to do it. For, randomisation is almost a *yogic* process involving the complete riddance, at least for the moment, of all one's obsessions, idiosyncracies, mental complexes and so on; in other words, the annihilation of the superstructure of Maya that envelopes us, holds us in its grip and sways us this way and that as it pleases. God is the great Randomiser and if we can randomise ourselves perfectly, we become one with God.

One wishes to try to be impartial in one's dealings but is not able to avoid partiality completely. Or again conversely, suppose one wants to organise something according to a certain plan, then chance elements somehow creep in and the plans have to be modified consciously or unconsciously. In carefully planned designs there are found elements of undesign or want of design while, in purposely

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[1] *This is slightly amplified from the original copy.*

purposeless acts like drawing random lines and writing down random digits, there are consciously or unconsciously some designs.

The statistician's occupation in life is to separate the elements of design from those of undesign. He has built up for this purpose a big edifice, the science of Probability, which deals with the measure of design in undesign and vice versa.

In the Bhagavad Gita, Chap. 18, verses xiv–xv, it is said:

अधिष्ठानं तथा कर्ता करणं च पृथग्विधम् ।
विविधा च पृथक्चेष्टा दैवं चैवात्र पञ्चमम् ॥

शरीरवाङ्मनोभिर्यत्कर्म प्रारभते नरः ।
न्याय्यं वा विपरीतं वा पञ्चैते तस्य हेतवः ॥

'Five elements termed causes account for the success of any action or undertaking which may be just or otherwise: first position, second the directing agency, third machinery of various sorts, fourth human labour and fifth (a sort of fifth column factor) chance'! This chance is the residual divine factor whose operation transcends human efforts of understanding. The statistician too does not understand how it operates but has devised some methods to find out and measure the effects of chance making use of artificially generated chance factors which are measurable. Just as you can measure one unknown length only with the help of another known length, you may measure one unknown chance factor with the measuring rod of known chance factors. For example, we know the chance of head or tail in tossing a coin, the chance of a certain number turning up in playing with dice, the chance of getting the ace of diamonds in playing with a complete suit of cards and so on. The statistician utilises these known chances in measuring unknown chances. Popularly speaking, chance is something that knows no law. This is a negative mark of no use to a statistician. A more fruitful way of looking at chance, the way that is exploited by the statistician, is to note that by chance all possible alternatives should occur according to a definite mathematical or theoretical frequency. In 1713, James Bernoulli stated the famous theorem that in the long run, all events will tend to occur with a relative frequency proportional to their objective possibilities. Nature seems to offer equal scope, opportunity and facility for the occurrence of all possible ways in which an event can happen, though she occasionally shows a bias in favour of one or more of them. But for this randomisation principle in Nature it would be impossible to build up

the science of Probability. As it is very difficult to separate completely the elements of design from those of undesign, partly because these sometimes simulate each other, the statistician is the only person competent to effect this separation as far as it is possible and we have to depend on his decision in regard to randomness. 'Random' is therefore what a competent and certified statistician certifies as **random.**

Now to come to our 'Random Lines'. If I have to test whether you have drawn them at random, I must first of all know what are the various types of these triplets, more properly speaking, your trilinear signatures. According to intersections, there are five major varieties: (i) nil-punctual, (ii) nni-punctual, (iii) bi-punctual, (iv) tri-punctual and (v) singular. There are subvarieties according to the limiting forms that the lines may assume when two of them which do not intersect are allowed to meet under certain conditions.

Two intersecting lines may form a cross (×), a semi-cross (ʏ), or a quarter-cross (<). The semi-cross carries with it an orientation suggested by the analogy of force-moments in mechanics. The bi-punctual class is exceptional; here the end semi-crosses are not oriented while the quarter-cross and the central semi-cross are oriented. Two non-intersecting, non-parallel lines are said to be skew when they can be made to intersect each other only by altering their initial characters with respect to the third line. The structure of a trilinear configuration is best studied by distinguishing their fundamental characters or dispositions into two classes:—(1) Interlinear disposition, which relates to two lines only, e. g., intersection, parallelism; (2) Intralinear disposition, which refers to a property of two lines with reference to the third line, e. g.. skewness. These dispositions or characters are invariant for the movements of translation and rotation. Two configurations having the same characters may be considered equivalent. The magnitudes of the angles, the lengths of the line-segments and the general orientation are not significant.

The interlinear disposition between two non-intersecting, non-parallel, non-skew lines is said to be symmetric, when both the lines have to be produced to meet each other, while, it is said to be asymmetric, when only one of them needs must be produced to meet the other. Hence the limiting form of a symmetric disposition is the quarter-cross, and that of an asymmetric one is the semi-cross.

We postulate that the symmetric and asymmetric dispositions alone can form limiting dispositions. The others are to be left

intact in forming limiting cases. Again, we assume that the bi-punctual variety is so oriented that the central line which meets the other two lines is horizontal.

Omitting impossible structures and taking due note of duplicate cases under the above conditions, we count the total number of possible varieties as 273.[1] By comparing the frequencies of occurrences of the actual types produced by you with the 273 possible types, the randomness in the performances can be assessed. We apply the so-called X^2 – test of significance and note the degree of probability in the results taken as a whole. Here, I must utter a caveat. Though I can tell 'en masse' whether your lines are really random, I cannot, for the life of me, nor can any other statistician for that matter, tell you individually, by any statistical canons, whether your performance is random or not.

Let me now pass on to random digits. They are a set of property-less things with no individuality, so to speak. If anybody perceives a lurking law in them, they immediately lose their randomness. They are a perfectly irregular class of numbers. But this irregularity has one regular feature about it, namely, all possible kinds of regularity must occur more or less with the same frequency. For a long time, it was thought that the rotating drum and the shuffled cards were effective means of randomisation. According to Karl Pearson, the records of whist, even those of long experienced players, show how the shuffling is far from perfect, and theoretically correct whist returns can be secured only by dealing cards without playing them. Tickets and cards, balls and beads have all failed in large-scale sampling tests, as for example, when you have to sample a cargo of coal or barley.

Once upon a time Mr. Tippett was struggling with a cart-load of tickets in connection with a sampling problem. He was then advised by Karl Pearson to invent a single random system of numbers, ranging from 0000 to 9999. Tippett went to that ocean of numbers, the census-reports, and by churning them, produced a set of numbers, which are now fairly popular in the statistical world for sampling purposes.

An ancient Indian legend records the churning of the Ocean of Milk, for the apparent purpose of obtaining Amrita or ambrosia, the panacea for mortality. The ancient churners never dreamed that they were

[1] This number may remind you of the *absolute scale of temperature* (Physics).

out on a randomising expedition, to assess the wealth of the ocean by random sampling. Just look at the entities that turned up. The essence of randomness was there, for after one entity emerged and showed itself up in the churning process, no one could say what the next would be like. Note the random sequence. First, the Moon, which proved later to be a satellite of the earth. Then, the great Goddess Lakshmi; next, an intoxicating drink, Wine; afterwards, a zoological specimen, the greatest horse in the world, Ucchaisravas; followed by an invaluable gem, Kausthubha, appropriated by one of the Trinity for Himself; later, a medical genius Dhanvantari, bearing a pot of nectar; succeeded by a great elephant Airavata, of huge body and huger tusks; and finally, a chemical, the worst of poisons, Kalakuta, capable of stupefying all the worlds, which another benign member of the Trinity swallowed for the safety of the worlds. (Mahabharata, Adiparva, Adhyaya 18). These were the products of a draw after powerful mixing up of the miscellaneous wealth of the ocean, and the random sample thus obtained gave a measure of the character of the contents of the great ocean.

As remarked already, Tippett was the first churner of that ocean of numbers, the census-reports, and obtained from it random numbers to the tune of 41,600. The census-reports have thus, for once, fortunately been able to justify their existence by the scientific purpose that they served in a manner that could never have been dreamt of by the administrators originally responsible for the great compilation.

After Tippett several churners entered the arena. If Karl Pearson be deemed a statistical Vasishta, then R. A. Fisher is his great rival Viswamitra, who always came up with rival schemes of statistical doctrine. So, Fisher made his own set of random numbers by resorting to another more modern ocean of numbers, the 20-figure logarithmic table of A. J. Thompson. By applying all kinds of random selections to this set of numbers, he produced his set of fifteen thousand numbers for the edification of himself and his disciples. Recently Kendall tried to construct random numbers from the numbers of the London Telephone Directory, a small lake of numbers compared with the oceans aforesaid. This proved, however, entirely unsatisfactory. He then constructed a randomising machine with a disc bearing the numbers 0—9 on the rim and rotating at a constant high speed. By illuminating the disc at random intervals by an electric spark or the flash of a neon lamp, numbers can be picked up from the apparently stationary disc by means of a pointer

fixed in space. In all these cases, though the methods involved randomness, the numbers derived therefrom had to be tested again for randomness and readjusted so as to satisfy the tests. Thus, in Fisher's table there was a bias towards sixes which were preponderant. This was squared up by replacing each of the necessary number of sixes by one of the other digits selected at random.

Tippett's and Fisher's sources for random numbers were not certainly numbers written out without any design. If the census-report-figures be called random digits we will be ungrateful to the hardworking enumerators, while if the log-tables should be deemed to contain random digits, we shall be insulting all the mathematicians of modern times. Why should we not, therefore, construct more random numbers from less random numbers? So I have ventured to construct random numbers by asking all and sundry to give me random numbers, which I will randomise still further with my randomising sieve. Thus, my method will randomise from start to finish in a progressive scale of randomisation.

Well, what is the use of these random numbers, you may ask. As I have said at the outset they replace effectively the rotating wheel, the shuffled cards and the blind dice-throw. For example, for lottery purposes you may make use of random numbers from Tippett's tables instead of a blind man's hand picking a numbered ticket from a randomised urn. It is very difficult, however, to construct a perfectly random set. There must be a perfect democracy of numbers, any two numbers being as unrelated to one another as any other pair. There should be no speciality about any of them, no freaks and no patches.

The principle involved in randomisation (i.e., avoiding all kinds of bias) is also the basis of all morality, all religion and all political ideals. What is morality except 'loving thy neighbour as thyself'? What is religion but to do justly, to love mercy and to walk humbly with thy God? What is your political ideal but to secure political unity with independence for each unit? Therefore I should say to every aspiring soul "Randomise yourself". Let us think our individual self as one of many having like properties or no properties. In the eyes of God we are like random digits or lines, one person or personality being as good or as bad as another. We are equally qualified or disqualified to receive the Divine Grace. God the Great Randomiser be with us and help us in our randomisations!

DISCUSSION ON THE TEACHING OF MATHEMATICS IN SCHOOLS AND COLLEGES

Chairman : Dr. F. W. LEVI

Opening the discussion Prof. G. R. GUNJIKAR (Ahmedabad) said that in the earlier stages the approach should be intuitional rather than rigorous and that teaching should follow the order in which ideas were developed by the human race. Too early an emphasis on rigour would choke the enthusiasm of the student. He was for a first course in calculus founded on graphical and physical experience before "analysis" was introduced. This would lead to a better understanding and appreciation of Analysis than is usual at present.

Dr. V. RAMASWAMI (Guntur) remarked that in discussing the teaching of Mathematics, it would be of help to know how the subject was taught in countries like Germany and France and requested the President and others present to recount their experiences.

Dr. LEVI (Calcutta) in response, said that until very recently, there were in Germany 3 different types of schools preparative to the University ; the main subjects where either classics, or modern languages, or mathematics. In the schools of the classical type, no calculus was taught, but he did not feel that students coming from these schools (as e. g. himself) were much handicapped when taking up mathematics at the University. Felix Klein took a keen interest in the improvement of the teaching of mathematics and he made large sections of the public conscious of its problems. His lectures on "Elementary mathematics viewed from a higher point of view" highly influenced two generations of mathematicians. An attempt with the same aim, but in a slightly different direction was the well known "Encykcopadie der Elementarmethematik" by H. Weber & F. Wellstein. [The speaker took the opportunity to express his gratitude to his late master H. Weber]. Under the influence of Klein, a national committee for the advancement of teaching mathematics was formed which became the nucleus of an international committee, the IMUK.* The reports of the IMUK contain a huge amount of interesting material and many practical suggestions ; they show the complexity of the problem. The work was stopped in 1914. Klein criticised the "double discontinuity" in the teaching : Coming from school (at the age of about 19 years) to the University, the student is confronted with problems which have no visible connection with the Mathematics he has read before ; returning from the University to school as a teacher, he is not able to apply his knowledge of higher mathematics. Klein's medicines were : *Fusion*, *Modernisation* and *Application*. "Fusion" means the union of the diverging branches of mathematics into a unified

* Internationale Mathematische Unterrichts-Kommission.

science; not only should analytic and synthetic geometry be taught as a single subject, but the links between geometry, analysis, algebra etc. should be made apparent by the teaching. Moreover Mathematics should be taught as a *living* subject and in the spirit of modern developments (this spirit has essentially changed since Klein's times); for this, the teaching of calculus was a minimum requirement. Finally Mathematics should be understood as an applicable science, essentially fostered by its applications. Though Klein's personality overcame the forces of inertia opposing his reforms, they met with material difficulties: The technicalities of school organisation restricted the application of the principle of fusion; calculus made "rigour" an educational problem where it hardly existed before and the emphasis given to the applications produced an avalanche of new subjects, practically working against the principle of fusion. Thus the reforms were not a full success, but they made the educationist conscious of the complexity of the problem and produced a better type of teachers. Educational questions were a permanent item at the meetings of the German mathematicians till recently; the Jahreshericht D. M. V. contain interesting reports of them.

Continuing, the speaker expressed the opinion that for the aims of general education through the medium of Mathematics, the selection of the topics is of less importance than the method of teaching. The pupils should be made to feel solid ground wherever they may be. But there were other purposes for teaching mathematics. He mentioned two items which he considers to be weak points with students of this country: the facility and reliability in numerical and simple algebraical calculations and the development of a true sense of spatial order.

Dr. V. RAMASWAMI (Guntur) said that it was necessary that the teacher should state clearly at each stage what it was that he was assuming and what he was proving.

S. SURYANARAYANA IYER (Trichinopoly) said that rigour was necessary in college teaching. He found that even in the honours classes there were students who did not quite know the rationale of the process by which the greatest common factor of two polynomials was found. Some of the text books in use like Smith's Algebra used in college teaching contained loose statements of results without all the conditions properly stated.

Dr. R. VAIDYANATHASWAMI (Madras) said that at each stage the learner could be expected to take certain things for granted and to be sceptical about others. Thus at the Pythagorean stage the existence of $\sqrt{2}$ was obvious, while at the Kronecker stage it was quite natural to ask "Can $\sqrt{2}$ be a number?" Logic develops with the individual and all that can be demanded is that mathematics should be rigorous at every level of logic.

"Time," he said, "is nearer our consciousness than space. Time is our friend and space our enemy and a general statement of the purpose of education would be that it aims to conquer our enemy space—an appearance

which mathematics seeks ultimately to master—by weapons of which the most effective is the real number concept. The study of vectors, geometrical forms and numerical calculation are processes by which this mastery is attained."

Prof. A. A. KRISHNASWAMI IYENGAR (Mysore) stressed the need for standardization, referring, in particular, to the sign of the perpendicular from a point on a line—a topic which was treated diversely by different authors.

Dr. B. S. MADHAVA RAO (Bangalore) felt that at the honours stage mathematics should be quite rigorous. Even M. A. students did not have clear conceptions of functional relations, and were unable to draw simple curves like $y = \log x$. He made out a strong plea for greater attention being paid to applied mathematics in colleges.

Prof. M. V. JAMBUNATHAN (Mysore) said that there are different levels of logic and consequently different degrees of rigour, and this fact should be borne in mind while considering rigour in the teaching of Mathematics. It will be very useful if students are encouraged to frame questions of their own and suggest enquiries of various kinds and to get satisfactory answers for them by self study or joint discussion under the guidance of the teacher.

Dr. P. S. SUBRAMANYA SASTRI (Annamalainagar) suggested the advisability of using the English technical terms in all linguistic areas so that those who take to higher courses of study might not feel any difficulty. This conference, he said, might take into consideration how elementary mathematics was taught in high schools. Overdoing of vulgar fractions and decimals in the lower secondary stage creates a disgust in the minds of more than half the students. The "concentric system" used in the high school classes reduces the interest of the boys in the subject. The teaching of rigorous geometry in the high school course will give a very necessary discipline to students to fit them for higher studies.

Winding up the discussion, the Chairman Prof. LEVI said that the use of the vernacular equivalent or of the technical term in English was one which was not peculiar to mathematics and was perhaps best discussed at educational conferences. He expressed the gratitude of the Indian Mathematical Society to the Founder, the Vice-Chancellor, the Registrar and all the authorities of the Annamalai University for the splendid arrangements they had made for the success of the Conference and paid a special tribute to the volunteers for their enthusiasm and helpfulness throughout the session.

Business Meeting of The Indian Mathematical Society

Held at Annamalainagar on 29—12—1943 at 9-30 P. M.

Chairman: Prof. F. W. Levi, President of the Society.

1. The Society paid its tributes to the memory of two of its departed members, the late Prof. P. K. Kashikar of Wilson College, Bombay and the late Mr. M. P. Khareghat of Malabar Hill, Bombay, all the members standing.

2. The Society placed on record its appreciation of the valuable services rendered to it by Dr. R. Vythyanathaswamy as President of the Society during the period of his office (1940—42).

3. The Society requested the President to convey its thanks to the Founder Pro-Chancellor, The Vice-Chancellor and the other authorities of the Annamalai University for their kind invitatation to hold the 13th Conference of the Society at Annamalainagar and for their generous hospitality during the session.

4. The question of reconstituting the Editorial Board of the Journal was discussed. It was resolved to request all the members of the existing editorial board together with Dr. S. Chowla (Lahore) and Dr. V. Ganapati Iyer (Annamalainagar) to serve on the reconstituted Board.

5. At the suggestion of the President, it was decided to appoint a special Publication Committee consisting of Prof. Levi, (convenor) Dr. Vaithyanathaswami, Dr. M. R. Siddiqi, Dr. A. Narasinga Rao and Dr. T. Vijayaraghavan with power to co-opt others, to collect funds and to arrange for the publication of monographs on special topics and of treatises.

6. The question of registering the Society was discussed and the advantages and difficulties in such a course pointed out. It was resolved to await the advice of a legal expert before taking any decision in the matter.

GLEANING

A moon, a nightingale and Her—
How different on Jupiter!
Nine moons across his heaven dance—
What invitation to Romance,
To nightingales and their sweet plaint
(If there were any,—which their ain't)
To maids aglo with looks aslant
(If there were any,—which there aren't).
Nine moons, No nightingales, No Her—
How terrible on Jupiter!

THE MATHEMATICS EXPOSITION

The Mathematics Exposition which marked out this conference from previous ones, was organized by the mathematics department of the Annamalai University with the cooperation of other departments of the University, many Indian Universities, Departments of the Government of India, publishing houses in India and abroad and several individual mathematicians. It was intended to give a concrete idea, as far as it could be done, of that rich and variegated legacy of intellectual experience accumulated through the ages which we call "mathematics", and the large part which it plays both directly and indirectly in determining the pattern of modern life.

The "Exposition" was arranged in the Arts and Science blocks of the Annamalai University buildings, and covered 11 rooms of which however, the last was devoted to spectacular experiments and displays of an electrical character.

Entering the first room which was devoted to MATHEMATICS IN INDIA, the visitor faced a galaxy of Indian scientific talent in the form of big sized portraits of all the 8 Indian Fellows of the Royal Society. At the centre was an oil painting of S. Ramanujan done by Sri S. Krishna Rao of Kumbakonam, with Sir C. V. Raman on one side and Dr. H. J. Bhabha on the other. In continuation, along the wings were the portraits of Dr. J. C. Bose, Dr. B. Sahni, Dr. K. S. Krishnan, Dr. S. S. Bhatnagar and Dr. M. N. Saha. Beneath each portrait were a few lines stating briefly the special contribution to knowledge which had secured for him a niche in the Temple of Fame. The famous "note books of Ramanujan" in original, were kindly lent by the Madras University and were exhibited in a glass showcase, while a sketch of his life and interesting results discovered by him were displayed prominently in the room: Below Ramanujan's portrait were the two quotations mentioned in Sri S. V. Ramamurti's speech and also the water colour depicting Bhaskaracharya pointing out to Lilavati the beauty of the Ganitapankajā (vide p. 74).

Pictures connected with the Indian Mathematical Society naturally had a place of honour in this room, among which may be mentioned a photograph of the Fergusson College Poona, the Headquarters of the Society, coloured photographs of the Founder V. Ramaswami Iyer and of the Foundation Editor M. T. Naraniengar framed in silk, of all the past Presidents of the Society, and group photos taken at previous Conferences. The Founder's letter dated 4th April 1907 announcing the formation of "The Analytic Club" as it was then called, list of foundation members, some of the early publications of the Society were among the documents connected with the foundation of the Society which were on show, along with several folio note books in the

handwriting of the Founder and containing his investigations in mathematics. A history of the Calcutta (1908) and Benares Mathematical Societies (1918) and copies of their latest proceedings were also put up as exhibits. The photographs of the two Indian Senior Wranglers, Dr. R. P. Paranjpye and Mr. A. T. Rajan were also on show with information regarding their contemporaries and their later achievements, the results of the Tripos that year etc.

Among the exhibits dealing with ancient Indian mathematics were imaginary pencil sketches of 6 mathematicians—Aryabhata, Brahmagupta, Varahamihira, Sripati, Mahavira and Bhaskara together with some of their best contributions in modern notation, and also an account of the Kerala school of mathematics.*

A number of books and manuscripts by ancient Indian Mathematicians, an astrolab ‡, a circular disc correlating the elements of the Indian panchanga† made up the remaining exhibits in the room. Running round the walls were quotations from classical works in Dravidian languages in praise of Mathematics, while on the blackboard was written out the following sloka from the Vedanga Jyotisha along with a translation :

यथा शिखा मयूराणां नागानां फणयो यथा ।
तद्वद्वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥

Passing on to the room dealing with the STORY OF MATHEMATICS, the visitor's eye was drawn to a series of pictures showing the red Sun of Mathematics rising over the horizon and illuminating a new concept which was later destined to be a landmark in the history of Mathematics. The country where the first glimpse of the new idea was caught, the immortals who were vouchsafed this vision, and its approximate date were all incorporated pictorially in these " mathematical dawns " of which there were 7 relating respectively to the invention of zero, of irrational numbers, complex numbers, algebraic geometry, calculus, non-euclidean geometry and relativity. On the opposite wall were a series of five linear time-scales each about 25 ft. long with the leading events and names marked to scale up to a certain point, after which the small remaining portion was coloured differently. It was this rump which was expanded into the whole of the 25 ft. of the next time scale, which was again marked in detail up to the point after which another application of the microscope was felt to be necessary. Thus the first time chart represented the period from 12,000 B. C. (first appearance of *Homo Sapiens*) to the present day marking therein the dates of the Babylonian and Egyptian calendars, the building of the great pyramid, Chinese mathematics and Thales of Miletus, the last portion covering about 2,500 years being expanded into the whole of the next chart. The second chart continued the Story of Mathematics in detail

* Kindly lent by Prof. A. A. Krishnaswamy Ayyangar.
† Kindly lent by Prof. A. A. Krishnaswamy Ayyangar and Dr. A. N. Singh.
‡ Kindly lent by the Archaeological Department of the Govt. of Mysore.

through the golden age of the great Greek masters and the Indian Mathematicians Aryabhata and Brahmagupta, the period from 640 A.D. to the present day being left over to be expanded into the third chart. In this chart which opened with the dark period which preceded the renaissance we find the great muslim mathematicians keeping the torch of mathematics burning. Alkhowarismi, Abu Kamil, Abul Wefa Omar Khayam while among those of other faiths we have Mahavira, Gerbert, Bhaskara Fibonacci and Roger Bacon. The fourth chart carries the story in detail from the renaissance (1450) to the Golden Age of Mathematics (1800—) and is marked with the names of Regiomantus, Leonardo da Vinci, Copernicus, Ferrari, Tartaglia, Vieta, Galileo, Kepler, Napier, Mersenne, Pascal, Newton, Euler, Lagrange, Gauss and Monge. In each case the main subject or subjects of study were marked along with the name of the mathematician, his position in the chart corresponding roughly to the period of his most important discovery rather than the year of his birth.

The last chart dealt with the Golden age of Mathematics (1800—) beginning with Laplace and Legendre and covers the period of the great makers of Analysis like Fourier, Cauchy, Weirstrass, of Geometry like Lobatchevsky, Bolyai, Riemann and of more modern mathematicians including those still happily in our midst.

Likewise, the mathematical time scale was shown in the form of a clock with the different epochs marked therein to scale.

Other exhibits of interest in the same room were: the genealogy of the Bernoulli family with the achievements of each of its members, a series of 5 pictorial representations* of the history of Mathematics, exhibits illustrating the evolution of the number system and of mathematical symbols, the numerals as written in various languages including one containing Tibetan numerals written by a Tibetan Lama on Tibetan paper.† At the centre of the room were placed the autographs of over a hundred leading mathematicians of all countries the bulk of which were lent by Prof. Levi, the photographs of many of these mathematicians,‡ a copy of Mathematical Reviews with details regarding the Microfilm Service, a Microfilm reading machine, and information about the Institute for Advanced Study at Princeton. The unsolved problems listed by Hilbert at the Paris International Mathematical Congress were exhibited on a big placard with blank spaces in which delegates were invited to comment on these problems in the light of present day knowledge. The blackboard in the room was used to give a list of the leading mathematical societies in the world, the year of their foundation and their official publications.

* Kindly lent by the Maharajah's College, Mysore.

† Kindly sent by Prof. F. W. Levi

‡ Kindly lent by Dr. A. N. Singh

Passing on to the room dealing with THE MYSTERY OF NUMBERS AND SYMBOLS, the visitor was informed that "Numbers rule the Universe" and that "God made the Universe, all else is the work of man" and was invited to pay his homage to a million dot rectangular lattice with intervals of 1/10 inch reigning enthroned over an entire wall of the room. Exhibits connected with Arithmetic included "Numerical giants", "Prime numbers" (including a note on primes, the biggest prime number known, and several others from the stratosphere of primes), "Fermat's Numbers and Fermat's last theorem,' "Mersenne's Numbers", "Pythagorean numbers," "Demlo numbers," etc. Several numerical curiosities adorned the blackboard, while others such as "a property of the number 30", "digits in the recurring period of $1/p$, $2/p$ etc" were placed on the walls, along with curious methods of multiplication (left to right, galosia, Russian method), a note on various methods of representing numbers, and tests of divisibility. Nomograms for the solution of quadratic and cubic equations competed with the Hydrostatic equation solver. The numbers e and γ to about 300 places of decimals in bold type ran along the walls while beneath them were placards about these numbers, as also an analysis of the frequency of the occurrence of the digits 0, 1, ... 9 in the decimal representation of e. The formula $e^{i\pi}+1=0$ in big golden letters was placed prominently on the wall. In General Algebra the topics selected for abstract postulational definition included the equivalence relation, closure, group, ring, field, linear sets, algebras, and partially ordered sets. The development of the real number concept, "which runs like a spinal chord through the whole of mathematics furnishing a solid support to its abstract structures and a centre of reference to its boundless perigrinations",* was illustrated in 4 ascending rectangular steps or stages: (i) The order-based cardinal numbers with set and the inclusion relation as basic concepts the structure being that of the partially ordered set, (ii) the group-based natural numbers closed for addition and its inverse operation, (iii) the rational numbers with "the field" as their mathematical structure, and (iv) the real numbers with continuity as the basic concept the mathematical structure being topological. A note on continuous functions without a derivate and other pathological functions, and a set of diagrams illustrating the first 6 stages in the construction of Hilbert's space filling curve were also exhibited in the room. Exhibits relating to Trigonometry included a teaching model for heights and distances, and details regarding the trigonometrical survey of India with placards illustrating various methods of map projection.

Geometrical models were kept in a room headed THE BEAUTY OF FORM. Among configurations in plane and in space were diagrams illustrating the notable points and circles of a triangle, and the Miquel-Clifford configuration, the Desargues (10,3) configuration of two triangles in perspective, Hesse's (12,18) configuration of two quadrangles in cyclic

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* Dr. Vythynathaswami; Presidential Address Aligarh Conference, *Math. Student* X p. 16.

perspective, the appollonian circles touching 3 circles, complementary Hart tetrads, a schematic representation of the incidences of the 27 lines on a cubic surface, and Schlafli's double-six. About a hundred graphs of algebraic and transcendental curves including some "algebraic ornaments"° and string envelopes adorned the walls. A number of wire and celluloid models of surfaces of the second degree, and about 50 wooden models of surfaces of special interest such as Enneper's surface, cubic cone, wave surface, helicoid, surfaces of constant breadth † were also on show. A number of glass models illustrating crystallographic forms were kept in a show case while the photographs of stellated and semi regular polyhedra taken from Max Bruckner's book "Vielecke and Vielflasche" were also exhibited. Besides the 5 regular solids in 3-space and diagrammatic representations of their groups, were also placards regarding regular solids in higher dimensional space. There were also several topological models of knots, one sided surfaces, † etc.

Three "Villas" were marked off in this room named respectively after Euclid, Lobatchevski-Bolyai, and Riemann and contained models of a cylinder, a pseudosphere and a sphere with a geodesce triangle drawn over each and placards giving the results in the three geometries, and pictorial material relating thereto from Lieber's "Three Moons in Mathesis".

The famous problems of trisection of an angle, duplication of a cube, squaring the circle and the four colour problem has a corner for themselves, each complete with its history and illustrative pictorial material. The publications of some modern angle trisectors were also exhibited to show that they were a hardy race. The various results achieved in connection with the 4 colour problems in the plane, and illustrations of the chromatic scheme for an anchor ring, projective plane were also shown as also a solid model to show that in 3-spac an infinite number of solids can exist of which each touches all the others, along an area. Visitors could see with their eyes the properties of conical projection in an apparatus in which a small electric bulb cast a shadow of a figure drawn on a glass plate on a plane below. The apparatus enabled the bulb and the plane to turn so as to leave the shadow unaffected till conical projection merged into plane perspective as a limiting case.

There was also a similar model for stereographic projection with a bulb inserted into the neck of a sperical flask so as to lie on the spherical surface produced, the shadow being caught on a plane below.

The room dedicated to THE WONDERS OF THE HEAVENS opened with a list of 25 makers of Astronomy with their respective countries, and dates of brith and portraits where available. Placards gave a survey of man's changing concept of the universe from the flat earth to the expanding Universe

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° Kindly presented by *Scripta Mathematica.*

† All these models were kindly lent by the Central College, Bangalore.

which was illustrated by ink spots on an inflated bladder. There was also a globe illustrating the Puranic concept of the Sapta Dwipas and Mount Meru.* The flattening of the Earth due to rotation was illustrated by a rotating steel band, while the precessional and nutational motions were illustrated by a gyroscope maintained electrically. On the blackboard was a big figure showing the orbits of Mercury, Venus, Earth, Mars and of Halley's comet with its positions during several decades marked therein. A number of perpetual calendars (vide pp. 7–9 of vol. XI of the *Student*), a siderial clock, a Bear and Cassiopeia Clock (to tell the time from the position of these constellations in the heavens) were on show. Placards connected with the distribution of stars in the Galaxy, various theories of stellar evolution and on the proper motions of stars summarised the latest knowledge on these topics. A star globe, several maps of the planetary and stellar systems published by Messrs. Johnstone were other exhibits, while star shaped holes in black paper pasted over a window pane and viewed against the background of the sky gave realistic representations of familiar constellations. Among the instruments in the room were a 6″ Max Kohl Telescope, a sexant, a model of an altazimuth a Diens Meterograph °complete with case and notice regarding the reward for its return, a model of a seismograph along with seismic records and of magnetic disturbances. In an adjacent room another Dien's meterograph was exhibited under an air pump to show how it responds to changes of pressure in its passage to the upper air. In another room were shown lantern projections of various astronomical objects from about 250 slides including 10 moved by rackwork.

.The STATISTICS section commenced with various types of frequency curves, Galton's . Binomial apparatus, and a twodimensional frequency distribution by means of rectangular wooden pieces. The application of statistical methods to weather prediction was illustrated by several charts kindly lent by the Meteorological Office, Bombay, while other charts exhibited the growth of populations, of index numbers of prices, and of prices of groundnut* (a local product). An analysis of linear triplets into 93 possible types (vide page 95), orthogonal latin squares,† a description of a randomisation machine † and details regarding the work of the Statistics Laboratory at Calcutta were other exhibits on show. A ruled table was placed in a corner and visitors were invited to help in determining the value of π by throwing the needle on the table and noting the number of occasions on which it intersected a ruling (Buffon's method). Two calculating machines ‡ (of which one was of the adding and printing veriety) and an addometer †† operated by the dial principle, were objects of considerable interest. Photos of eminent statisticians and copies of statistical journals were distributed over the room.

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* Kindly lent by Mr. S. R. Ranganathan, Madras.
° Kindly lent by the Meteorological Department.
* Kindly lent by Mr. S. Subramaniam, Delhi.
† Kindly lent by Prof. A. A. Krishnaswami Ayyangar.
‡ Kindly lent by the Maharajah's College, Mysore.
†† Kindly lent by Mr. T. S. Ramanujam, Madras.

The section on AMUSEMENTS AND RECREATIONS was naturally the most popular. On the walls were problems of Cryptarithmetic (arithmetical problems in whicn some of the digits were replaced by letters and had to be found out), various other arithmetical amusements and various possible knights moves on ordinary chessboard and on chessboards of different sizes and shapes with special conditions, taken from Kraithchik's book. A number of games were arranged on tables in the middle of the room and the visitors were invited to play the games with the volunteers in charge of them. These included.

The battle of numbers,	Nim and its generalizations
Bachct's weight problem,	Tower of Hanoi
Think of a number,	Chinese rings
Magic windows,	Hamilton's Dodecahedron problem
Jack o'nine	Josephus problem
Oil monger's problem	The 15 puzzle
The jealous husbands' passage	Mac Mohan's coloured cubes problem
The eight queens problem	The regiment's problem
Solitaire	The lazy warden's problem

In each case there was a placard explaining the problem, giving its history and its mathematical theory (where it existed) and its generalizations. Magic squares, cubes, pentagons, hexagons, circles, fractions were exhibited on another wall where the theory of the Magic square was also explained.

Books on Mathematical recreations by the following authors were also placed as exhibits to give further information and literature relating to the various problems: Ball, Dudeny, Ahrens, Kowelewski, Lietzmann, Rademacher und Tœplitz, Bachet, Lucas, Kraitchick, as also several issues of The Sphinx and the Comptes-rendus du Premiere Congress international de recreations mathematiques.

Among the applications of Mathematics to various fields of human activity, the section on MATHEMATICS AND ART gave an account of Birkhoff's Theory of Aesthetic measure and over fifty polygonal patterns from his book with the æsthetic measure of each marked underneath. The theory of plane filling repeating patterns was discussed in a placard while several coloured patterns consisting of regular polygons were exhibited. The connection of these patterns with Group Theory was also pointed out and illustrated by several border ornaments, linoleum patterns etc. The series of interesting pictures published and kindly presented by Scripta Mathematics entitled "Mathematical themes in ornament", "In a laboratory of design" and another set of "Algebraic ornaments" were on show here. The principles of perspective drawing were explained and illustrated by a model "perspective apparatus" in which the picture plane was represented by a wire mesh and rays of light converging at the artist's eye by strings. There were séveral optical illusions including a figure in Indian ink which looked at

steadily gave alternately the impression of a flower vase or of President Levi gazing admiringly at his own double. Another item in visual geometry was the apparent changes in the shape of a pattern on the ground like a triangular field when viewed through a railway window.*

MATHEMATICS AND WAR gave an account of how mathematical methods are used in modern warfare in Ballistics, sound ranging, prevention of wing flutter, airfoil sections, submarine design, navigation by sea and by air etc. In MATHEMATICS AND INDUSTRY were pointed out the applications of determinants, bicircular coordinates, continued fractions, elliptic integrals, mechanical equation solvers and other mathematical disciplines to industrial problems as given in T. C. Frys "Industrial Mathematics"† along with the diagrams given in the booklet. The use of mathematical methods in the design of gearing wheels and various branches of the automobile industry were illustrated from booklets kindly presented by Burma Shell Co.

MATHEMATICS AND BIOLOGY was a rich section with a room to itself in which the Zoology Department has arranged the following series of interesting exhibits:

The equiangular spiral and its properties with reference to the Molluscan Shell:

Section of a large number of South Indian marine and non-marine shells were exhibited to illustrate the variations of the principle of the logarithmic spiral. A series of specially selected shells, both entire and in sections, were exhibited with measurements to illustrate how the appearance of the shell is determined by the magnitude of (1) the constant angle of the equiangular spiral, (2) the enveloping angle of the cone in turbinate shells, or the angle which a tangent to the whorl makes with the axis of the shell and (3) the "angle of retardation"—which expresses the retardation in growth of the inner as compared with the outer part of each whorl. How these angles are in their turn determined by the ratios of certain velocities of growth was also illustrated.

It was shown that a moth, when flying to a light which is close to it, will move along a spiral ending at the light itself. The response of the insect to light depends on the maintenance of a constant pattern of stimulation on the retina. Owing to the peculiar structure of the insect eye, the angle of incidence of light can be kept constant, only if the insect continually turns towards the source of light.

Charts and graphs were exhibited to illustrate inward or physiological time, how "young and old, united in the same space live in a separate universe where the value of time is radically different". Appreciation of time by the honey bee was illustrated by a chart.

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* Vide A. Narasinga Rao: Through a railway window, Jour. Ind. Acad. Sc. Vol. p

† *American Mathematical Monthly* Vol. 48 No. 6.

The mathematics of the double row of cells of the bee-hive was illustrated with the aid of wooden models and the minimum surface area properly pointed out. Other kinds of hexagonal conformations like the coral, compound eye of insects etc. were also exhibited.

The mathematical principles underlying the form and branching of blood vessels were illustrated by suitable exhibits. It was shown in particular how the branching of an artery may be looked upon as a special case of "grazing incidence".

A large number of charts and diagrams were exhibited to illustrate the importance of the statistical hypothesis in biology, chiefly with reference to Mendelism and the modern theory of the gene. specimens of *Drasophilia*, giant chromosomes and sex chromosomes were shown under the microscope.

There were also some exhibits relating to the limitation of the size of animal forms and heights of trees in order that they may not collapse under the strain of their own bodily structures.

The Physics Department of the University had arranged a number of interesting experiments in the room dedicated to THE POETRY OF MOTION. These included experiments illustrating the combination of harmonic oscillations (Lissajou's curves and Blackburn's pendulum), Chladni's figures, Eigen-oscillations of various dynamical systems with 2 or 3 degrees of freedom (with a row of ballet girls going through the same movement as a pictorial illustration thereof) forced oscillations, stationary waves, wave motion on water illuminated by intermittent light and thrown on the screen to illustrate interference etc. The Experiments in elasticity included the study of strain in glass by polarised light, and the determination of Young's modulus by an optical method. Surface tension phenomena were illustrated by the aniline experiment, minimal surfaces through a given boundary in the form of soap films, and the motion of camphor on water. Other interesting exhibits in the room were Barker's mill, cartesian diver, Tantaulus cup and a double cone running uphill by gravity.

Mr. M. L. Sitaraman, Lecturer in Chemistry who has been making a special study of Vedic chants and their connection with bird music by means of savart wheels, arranged a number of interesting experiments including simple experiments with tuning forks on the limits of audition of the human ear. Yajur and Sama Vedic hymns reproduced with sets of toothed wheels maintained electrically in uniform motion (savart wheels), and the ratio of notes in Chinese music in the so called scale of fifths. Notes corresponding to our simpler intervals were all demonstrated with savart wheels.

As stated already, the last room was devoted to spectacular experiments of an electrical character amongst which were the production of electricity by motion, heat, or light with their applications (photo electric cell, burglar alarm). The discharge of electricity through gases illustrated by various

vacuum tubes (one of which was 8 ft. long). A rolling wheel maintained by an electron stream, and a motion of an electron beam round a magnet, were attractive exhibits. Barlow's wheel, and Waltenhofen's pendulums to illustrate eddy current, were other experiments in the room.

All along the walls of the several rooms at uniform intervals were placed portraits of eminent Mathematicians, physicists and philosophers from the three well known portfolios of such savants published by *Scripta Mathematica* and underneath each portrait was the descriptive sheet which goes with the portrait. There were also more than a hundred pictures of mathematical interest, which were also kindly presented by the Editor of Scripta Mathematica to the Conference. Each portrait was placed in the room which related to the activities of the person portrayed. Besides there were placed on the walls two sets of posters with heading "Do you know?" and "Brain Teasers". The former sought to draw the visitor's attention to little known but interesting material of a mathematical character, while the latter gave provoking little problems, or paradoxical statements or puzzles to stimulate thought. Some of these will be published as "Gleanings" in the pages of the *Student*.

The exposition included a BOOK AND REPRINTS SECTION to which authors and publishers of mathematical books contributed a large number of exhibits, mostly text books and reprints of mathematical papers. The Benares Mathematical society, the University of Calcutta and the Osmania University sent their mathematical publications for exhibition. Among those kindly presented to this section by foreign publishers and which attracted attention we should like to mention:

Scripta Mathematica Library Publications: 5 booklets

Keyser: Human worth of rigorous thinking.

Peacock: Treatise on Algebra 2 vols.

Huntingdon: Foundations of Algebra

Lieber: The Education of T. C. Mits

Lieber: Non Euclidean Geometry; Einstein's theory of relativity; Galois and the theory of groups (3 booklets)

Bell: Men of Mathematics,

E. Kasner and J. Newman: Mathematics and the imagination.

The Society is grateful to the several Indian Publishers, Indian Mathematical Societies, Universities, individual authors and to Messrs.: Burma Shell, Simon and Schuster, H. G. and L. R. Lieber, and J. Ginsburg, Editor of the Scripta Mathematica for their kind presentation of their valuable publication to the conference.

A. N. RAO.

REVIEWS

Recent Advances in Galactic Dynamics

By A. C. Banerji, M. A. (Cantab.) M. Sc. (Cal.) F. R. A. S., F.N.I. (Lucknow University Studies, No. XV, Lucknow 1942)

This interesting brochure is the outcome of a course of lectures delivered by the author at the Lucknow University in the year 1940, and contains a resume of the recent advances made in our knowledge of some of the outstanding problems of modern Astronomy. The title chosen for the monograph does not seem to correctly indicate the scope of the work; the topics selected for presentation are chiefly in connection with such diverse subjects as the internal structure of stars, the formation of arms in spiral nebulae and the birth of planetary systems, subjects to which the author and his co-workers in Allahabad have made contributions of some interest during recent years.

The first lecture which deals with polytropic configurations of gaseous bodies opens with an introductory sketch of the work of early investigators. This is followed by a discussion of Emden's equations for stellar configurations for a non-rotating state. The case of rotating compressible masses is next considered, passing in brief review the work of Roche, Jeans, Milne, von Zeipel and others. The investigations of Chandrasekhar and Kopal, assuming uniform rotational velocity are described in some detail, following which we have an exposition of Bhatnagar's work in cases of variable angular velocity. Reference is made of most of the recent investigations of the subject including Eddington's model of a polytrope with a variable index.

The mechanism underlying the formation of arms in spiral nebulae is the subject of the second lecture. The theories of Jeans, Brown, Lindblad and other workers are critically examined and a brief account is given of the investigations of the author and his colleagues on the conditions necessary for the formation of spiral arms, where it is assumed, following Plaskett and Pearce that the rotating spheroidal central mass of finite dimensions and uniform density is surrounded by a spheroidal structure of rotating compressible gas of variable density. The effect of ionisation originally pointed out by Eddington is taken into account in a subsequent investigation. It is however admitted that in the present state of our knowledge of the structure of galaxies, it is difficult to formulate a satisfactory theory regarding the formation of arms in spiral nebulae and the knots that are observed in them.

In the final section we have an interesting account of the various theories that have been advanced from time to time about the origin of the solar system and the birth of planets, a subject which has attracted the attention of scientific men for nearly two centuries. After giving a brief review of the earlier

theories, the recent researches of Lyttleton are described wherein it is assumed that the Sun was originally a binary star and the companion having had a tidal encounter with a third star, ultimately escaped away from the Sun. The objections pointed out by Luyten are mentioned in some detail, following which we have an account of the investigations of Bhatnagar, Spitzer and others which appear to lend support to the view that none of the existing tidal theories can fully explain the formation of the solar system. The chapter concludes with a discussion of the new theory propounded by the author, based on the assumption that the Sun was originally part of a Cepheid variable star and the near by passage of another star with about the same mass, produced large tidal disturbances and violently increased the Cepheid's oscillations; as a consequence of the dynamical instability thus set in, matter was ejected from the Cepheid, resulting in the formation of planets.

Although the treatment of some of the topics appears to be too brief, the author has succeeded in presenting a readable summary of the current ideas with respect to some of the outstanding problems of stellar astronomy. A fairly complete bibliography is given at the end of each section which will be helpful to the student desirous of obtaining more detailed information about the problems that are discussed.

T. P. B.

Elementary Trigonometry
Elementary Analytical Geometry
Elementary Calculus

By M. V. Jambunathan, M.A., Lecturer, Mysore University, Seshadri & Co. Balepet, Bangalore City.

The author of the above three text books has met with exceptional success in his aim, namely, to present the subject matter of these books in a clear and simple manner so as to rouse the interest of the pupils in the respective subjects. These books are intended to suit the requirements of the Intermediate examination of the Mysore University.

The book on Trigonometry begins with an interesting historical note on the contribution of Ancient Indian Mathematicians to Trigonometry. Details are discussed fully in the course of the book and the exposition is throughout lucid. The book abounds in worked examples and contains a good number of exercises.

The same high standard of exposition is kept up in the book on Analytical Geometry which contains a thorough discussion of the subject as applied to straight lines. A chapter treating the elementary properties of circles by analytical method would have made the book suit also the syllabus of the other south Indian Universities.

The author is to be particularly congratulated on the able manner in which the idea of a limit has been formulated and discussed in his book on Calculus. The notion of a limit, so fundamental in Calculus and higher

mathematics, is usually a stumbling block for the beginner. By varied concrete illustrations the author leads up to the general notion in a clear and easy manner. While the treatment as it stands is exceptionally good, it may be permitted to remark that it would be better to emphasize and to state bluntly in so many words that the limit of a function at a point is a property of the values of the function at points *other than* the value at that point. Till this point is brought home to the learner by examples and illustrations, his notion of a limit is likely to remain vague. Another point to be mentioned is that the book would have lost none of its good points if one or two paragraphs have been added to the chapter on the idea of a function emphasising the fact that a function is defined for a given set of values of the variable and *not for all* values as would be supposed by a student who reads the chapter.

The get up of all the three books are attractive and the prices (Rs 1-12-0, 0-12-0, and 1-4-0) are very moderate. V. G.

BOOKS RECEIVED FOR REVIEW

B. SEGRE: The Non-singular cubic surfaces; Oxford 1942.

Presented to the Library

STRUIK: Grundzuge der Mehrdimensionalen Differentialgeometrie

(Presented by Prof. N. S. Nagendranath, Andhra University, Guntur)

ANNOUNCEMENTS AND NEWS

The following gentlemen have been admitted as members of the Indian Mathematical Society:

Gopichand Patni Esq. M. Sc., Lecturer in Mathematics, Podar College, Nawalgarh, Jaipur State;

K. G. Ramanathan Esq., M.A., Research Student, Madras University, Madras;

H. G. S. Sharma Esq., M. Sc., 39 Dodda Mutt, Akkipet, Bangalore;

S. K. Gokhale Esq., M.A., Lohia Bazar, Lashkar, Gwalior;

S. K. Ghose Esq., M.A., near Kali Bari, Ambala Cannt;

T. V. Vasudeva Esq., M.A, Lecturer, Pachaiappa's College, Madras;

V. Seetharaman Esq., M.A., M.Sc., Lecturer, Annamalai University, Annamalainagar;

A. Suryanarayanamurti Esq., M.A., Research student, Madras University, Madras;

P. C. Mathur, Esq., M. Sc., Civil Lines, Fyzabad;

Abbas Hassan Razvi Esq., M.A., Lecturer, Osmania University, Hyderabad (Deccan);

Afzal Ahmad Esq., M.A., Lecturer, Osmania University, Hyderabad (Deccan);

We offer our hearty felicitations to Dr. R. P. Paranjpye on his appointment as the first Indian Representative to the Commonwealth of Australia. The first Indian Senior Wrangler, Dr. Paranjpye has dedicated his life to the cause

of education and later to politics. He is one of the earliest members of the Society and was its President from 1936 to 1940.

The Government's insatiable hunger for statistical experts is depleting the ranks of the teachers of statistics in educational institutions. Among the most recent to be thus transformed may be mentioned Prof. K. B. Madhava the organiser of an advanced school of Mathematical Economics and Statistics in the Mysore University and long considered an integral invariant of that University, who has been appointed Special Officer, (Population data), Education, Health & Lands Department, Government of India, Presidency College, Calcutta. Others subjected to the same transformation include Mr. N. Sundararama Sastriar of the Madras University who is now Statistician, Dept. of Labour, Govt. of India and Dr. B. Ramamurti of Ajmere College who is working as Progress Officer, Misc. Stores, New Delhi.

David Hilbert, Prof. of Mathematics at Gottingen and one of the greatest mathematicians of our time passed away in the summer of 1943 at the age of 81. Hilbert's contributions cover a wide range of mathematics — theory of invariants, algebraic numbers, foundations of geometry, calculus of variations, integral equations, Dirichlet's principle, the foundations of Physics and Mathematics and lastly Logic. In choosing him for the award of the Bolyai Prize (of the Hungarian Academy of Sciences) awarded to "the author of the best work in mathematics published during the last 5 years", Henri Poincare wrote in 1912 of Hilbert's works that they "par la prondeur de la pensee, l' originalite des methodes, la rigueur logique des demonstrations, ont deja exerce influence considerable sur l' progres des sciences mathematiques."

In the sudden death in June 1944 of Prof. Kombur Sesha Iyengar Kuppuswamy Iyengar (better known as K. S. K. Iyengar) at the age of 45, the Indian Mathematical Society has lost one of the most lovable and colourful of its members. After passing the Honours examination with distinction from the Presidency College, Madras, K. S. K. Iyengar spent 5 years in Cambridge where he came under the spell of the Hardy-Little wood tradition and developed a profound admiration for the continental work on the Theory of Functions. These remained abiding interests throughout his life and were responsible for the high standards of rigour which he maintained as Prof. of Mathematics at the Central College, Bangalore. At meetings of Boards of studies or examiners, he was one of the most well informed of members, quick to catch a point or resolve a difficulty whatever the topic of discussion. He has published over 20 papers on the problem of the tetrahedron, summability and Tauberian theorems, orthogonal functions, integral functions and on the mathematical aspects of the Bhabha-Heitler cascade theory. His range of interests extended far beyond mathematics and include Buddhism, Yogic Sadhan, religious mysticism and pedigree dogs (of which he kept some fine specimens). He was one of the most unconventional of men, frank, and outspoken and was universally respected and loved.

LIST OE PERSONS WHO ATTENDED THE CONFERENCE

Mr. S. K. Abhyankar, Professor, Gwalior.
" Afzal Ahmed, Research student, Osmania University, Hyderabad.
" Appaji Josier, 60, Rajah street, Coimbatore.
" K. Balagangadharan, Madras Christian College, Tambaram.
" K. Chandrasekharan, Research student, Madras.
" V. Ganesan, student, Engineering College, Guindy.
" K. S. Ganapati, Hindu College, Tinnevelly.
Dr. V. Ganapati Iyer, Annamalai University.
Mr. T. Govindarajan, Annamalai University.
" K. R. Gunjikar, Principal, Guzarat College, Ahmedabad.
" M. V. Jambunathan, Maharaja's College, Mysore.
" D. D. Kapadia, 6, Staunton Road, Poona.
" D. R. Kaprekar, Teacher, High School, Devlali.
" A. M. Krishnamurti, Lecturer, National College, Trichinopoly.
" R. Krishnamurti, Professor, Nizam's College, Hyderabad.
" V. S. Krishnan, Research student, Madras.
" A. A. Krishnaswami Ayyangar, Maharaja's College, Mysore.
" G. V. Krishnaswami Ayyangar, Annamalai University.
Dr. F. W. Levi, Calcutta University, (President).
" B. S. Madhava Rao, Central College, Bangalore.
Mr. S. Mahadevan, Presidency College, Madras.
" V. A. Mahalingam, Loyola College, Madras.
" T. Manickavachagam Pillai, Annamalai University.
Dr. S. Minakshisundaram, Andhra University, Guntur.
Mr. O. R. Muthuswami Aiyar, Sir Sivaswami Aiyar High School, Tirukattupalli.
" K. Nagabhushanam, Lecturer, Andhra University, Guntur.
" K. Nagaraja Aiyar, Advocate, Pudukottai.
Dr. A. Narasinga Rao, Annamalai University.
" K. Panchapakesa Aiyar, H. H. The Rajah's College, Pudukottai.
Mr. A. S. Puri, Kapurthala.
Rev. Father C. Racine, Loyola College, Madras.
Mr. S. Raghavachari, Principal, National Tutorial College, Kumbakonam.
Dewan Bahadur T. Raghaviah, Madras.
Mr. A. T. Rajan, Madras.
" S. V. Ramamurti, Adviser to His Excellency the Governor of Madras.
" V. Ramaswami, Andhra University, Guntur.
" K. Rangaswami, Annamalai University.
" K. Sambasiva Rao, Andhra University, Guntur.

Mr. L. Sanjiviraya, Cuddapah.
„ S. S. Sankaranarayanan, Madras.
Dr. S. R. U. Savur, Principal, Government College, Palghat.
„ V. Seetharaman, Annamalai University.
„ S. M. Shah, Muslim University, Aligarh
Mr. Shiersat, Deolali.
Dr. A. M. Shaikh, D. J. Sind College, Karachi.
„ A. N. Singh, Lucknow University, Lucknow.
Mr. M. V. Subba Rao, Research student, Madras.
„ K. Subrahmanyan, Hindu College, Tinnevelly.
„ L. N. Subrahmanyan, Madras Christian College, Tambaram.
„ S. Subrahmanyan, Statistitian to the Economic Adviser to the Govt. of India, Delhi.
„ A. Suryanarayanamurti, Research student, Madras.
„ S. Suryanarayana Iyer, St. Joseph's College, Trichinopoly.
„ A. Suryanarayana Sastri, Sir Sivaswami Iyer High School, Tirukattupalli.
„ T. Totadri Aiyengar, National College, Trichinopoly.
Dr. R. Vaidyanathaswami, Madras University.
Mr. M. Venkatarama Aiyar, Central College, Bangalore.
„ T. K. Venkataraman, Principal (Retd.), Kumbakonam.
„ T. S. Venkataraman, Union Christian College, Alwaye.
Dr. T. Venkatarayadu, Hindu College, Masulipatam.
Mr. S. Viswanathan, Cuddalore.

GLEANINGS

Aeronautics

The essential mathematical problem is the flow of air across obstacles and the determination of the surfaces of least resistance and greatest lifting power. The essential tool is conformal mapping.

One of the earliest used is the Jukowski transformation $Z=z+a^2/z$ most simple for two dimensional motions. It converts the circle into a suitable form for the section of an aeroplane wing.

Experiments are actually performed in a windtunnel on small models. The corresponding result in the actual air-plane is deduced by a mathematical reasoning.

Sub Harmonic Resonance

Consider the small oscillations of a body whose free period is $\frac{2\pi}{\omega}$ when there is a damping coefficient varying as the velocity and elastic restoring force which is a function of x. We have, $\ddot{x}+\mu\dot{x}+\omega^2x=\psi(x)$. We have here as observations show oscillations of period $\frac{2\pi}{\omega}$ whenever we have a periodic force whose period is a submultiple of the period of the body.

This is the explanation of the phenomenon of fluttering in aeroplanes, when, at high speeds the wings begin to vibrate so violently as even to wreck the machine completely.

From, *The Annamalainagar Mathematics Exposition.*

PRESIDENTIAL ADDRESS

WHY MATHEMATICS?

Presidential address delivered at the Thirteenth Conference of
The Indian Mathematical Society
held at Annamalainagar

BY

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Even in this age of restless work and incessant labour, thoughtful human beings have the privilege—and even the duty—to lay down their tools from time to time and to think over the true meaning of their task and of their work. Far from the focus of public interest, neither much encouraged by the applause of the masses nor disturbed by their criticisms, mathematicians enjoy an unusual intellectual liberty which implies the full responsibility for their attempts and for their achievements. We are enthusiasts of our science; we enjoy its harmonious beauty and the triumphant moments of successful thinking. Otherwise we could not be mathematicians. But are we, as responsible persons, justified in devoting our whole life and our full energy to these activities? Is mathematics done only for the enjoyment of its disciples, or is it a necessary part of human culture? *Why Mathematics?*

I could not raise this question at to-day's festive occasion without believing in the importance and necessity of mathematics, but I ask the audience not to take the question mark as a mere rhetorical figure, some doubts and scruples uttered at the beginning of the speech to let the subject appear even more radiant in an apotheosis at the end of the address. The question is asked in full earnest; the answer, far from being complete, is only an attempt to give some clues to what seem to me the foremost arguments justifying mathematics as a science and the principal reasons for the importance of its further advancement. For this investigation we have to try a descent to the depth where the roots of science are to be sought, but let us start from the surface.

In general a mathematician is as profoundly convinced of the importance of his subject as are e.g. painters, sculptors, musicians and actors. A painter, asked why he paints, may answer that he

feels compelled to do so and he may refer the questioner to the books on the philosophy of art stored in the libraries to gratify his theoretical curiosity. This attitude is sincere and respectable though it does not involve a satisfactory reply to the question. A mathematician has an easier way to face—or at least to escape—such a challenge. "Mathematics" he may say "is a complex subject, but its usefulness is evident by its applications. Without mathematics there would be no railways, steamers, motorcars nor airplanes. The production of our industrial age is based on mathematical calculations and the administration needs mathematical applications of statistical data. Thus mathematics is an essential prerequisite of modern economic life".—There is nothing untrue in a reply of this kind, but nevertheless it is insincere because it avoids the main-issue: the value of mathematics in itself. The boasting reference to the successes of applications of mathematics is a convenient expedient to avoid a superficial conversation with frivolous people about a serious topic, but even from a purely practical point of view there is some danger in it. It is not good to burden mathematics with the responsibility for modern life and all its consequences which show many embarrassing features. People may argue that the destruction of mathematics may bring about the fall of the whole structure of modern life which is based upon it. It is not so long ago that the progress of science and engineering made the public expect the approach of a golden age in the future, perhaps even in the near future. Philosophers and other observers who were horrified by certain cultural developments of that period, were disregarded, but in later years events and changes in political, social and economic life reduced and eventually shattered the hope of a golden age due to scientific and industrial uplift; again people looked back to a golden age of simplicity and safety at a time unknown to the historians, since history records only blood, sweat and toil—and the longing for a golden age. "Beatus ille qui procul negotiis ut prisca gens mortalium" (Blessed who is far from business as were people at some remote time); this ode of Horace is two thousand years old, but it touches even the non-believer in golden ages. How fortunate to live far from the trouble of economic life, far away at a lonely place. Man will find time to think of himself, to dive into his own soul and to salvage the treasures hidden there. He may become a saint or a poet or perhaps a mathematician. Mathematics existed and was appreciated long before there were motors and telegraph lines; it is not a child of the age of engineering though certainly the printing press, libraries and easy communications are greatly helping its advancement; we must be grateful for all

these institutions. But the only really essential thing for mathematics is the man and his mental power. In his own soul he finds the material to build up a new world, material as unpretentious and simple as e. g. the numbers and the notion of space.

The idea of number contains " plurality " as well as " order "; this so to speak inborn interconnection of two important notions in one, makes it particularly fit as a basis of further logical developments. The creative will of the thinking mathematician now intervenes and starts operating on the original objects, e. g. he might combine two pluralities into one by addition. By reversing the operation he gets a new operation, the inverse operation; other operations can be created by combining operations by a rule involving a certain order. Similarly the combination of the same operation with itself, the iteration, involves an index of plurality indicating how often the operation is repeated. As pluralities can be expressed by numbers, the iteration of an operation A can be considered as a new operation which depends on one parameter more than A. This " promotion of an index of operation to a numerical parameter " is a very important method; it leads to new problems at a later stage when the notion of number is generalised. A new and important step is taken when the operations are made new elements, the " operators "; then the combination of the former operations is considered as an operation on those new operators. In this manner the sphere of entities to be considered and of mental activities to be performed by the mathematician increases indefinitely. Every system created in this way creates an opportunity for a new system formed by the automorphisms of the first one, but the seed of this branch of mathematics is the idea of number combining in itself " plurality " and " order ". In this way the original idea of number re-enters mathematics at every stage of investigation. The objects of the operations may be different from numbers; they may even be freely created elements which have no meaning by themselves and there is no limitation for the selection of operations except that they have no self-contradicting properties. However the natural numbers are coming in of necessity as " pluralities " and " indices of order " so that when replaced at one place by generalising notions, they return so to speak at a higher plane. So mathematics shows the basic importance of the concept of numbers in the fabric of our mind; moreover it furnishes also the proper method to investigate the true meaning of " space ".

Although geometry has been treated through millennia, it is still an inexhaustible well of new ideas. Not only masses of special investigations are produced every year, but it seems that the problem of

space appears under an essentially new aspect to nearly every generation which is active in mathematics. There is no line of demarcation separating geometry from that branch of our science which is based on the notion of number; they penetrate one another and they have grown together like twin-trees which show only a single stem to the observer though they are springing forth from two separate seeds. Space appears to be an ordered system with a type of order fundamentally different from that of the natural numbers. There is no succession of discrete entities in space and no natural starting point; one is dealing here rather with a homogeneous and isotropic medium without gaps. Notions like point, direction and measure help to distangle this maze. The latter one furnishes the possibility for applying arithmetic. In doing so, one obtains numbers as "proportions" and this gives another, a geometrical, concept of numbers. Thus plurality, order and proportion are three pre-mathematical ideas from which one can start to obtain the notion of number. Mathematics is based on all three of them; the fact that the further development does not lead to logical difficulties which are more serious than those treated in the theory of the continuum, shows that those basic ideas can be considered to be compatible.

These few remarks show already how mathematics discloses the nature of the fundamental elements of our intellect, or at least of some of them. Simultaneously it amplifies ways of thinking—namely the mathematical ways—by all those methods which form the mathematical literature. An enormous display of intellectual force and dialectic skill, but not a justification of mathematics, unless mathematics is considered as a value by itself without any reference to anything else. If however one adopts this point of view, the question: "Why mathematics" has no sense. On the other hand we renounced a claim based on the value to be attributed to engineering and similar applications of mathematics. So we got the net result that mathematics is the outcome of some basic ideas of our mind which we feel compelled to investigate and which by the help of progressing mathematical investigation are proved to be interconnected in various ways. One could speak of the *introspective* value of mathematics. I do not suppose that any other intellectual method has reaped so many and so important results for introspection, but we should not forget or conceal the fact that these results are the harvest of millennia. However the introspective value of mathematics is not its only justification. We should reconsider the case and for this purpose let us ascend again from the depth of the

human soul and the seeds of the notions to the surface of life. Using the deductive method I shall shortly review the reasons why individuals and peoples deal with mathematics.

At all times and in every country, there is the "mathematics of the bazaar" the arithmetic of buying and selling, of counting coins, goods and transports. Through ages it has changed very little; only in modern times it was increased by subjects like actuarial science and economic statistics which sometimes apply advanced mathematical methods. Besides this commercial arithmetic, other types of mathematics have been developed in the centres of culture throughout history; they owe their origin to a great variety of different reasons. For instance the geometry of ancient Egypt is based on the needs of topographic survey. The mathematics of the Middle East is influenced in the first place by problems of astronomy; this holds for the period of the cuneiform characters as well as for the much later period of the Arabic mathematicians. The scholars of ancient India developed a mathematical science in which the theory of numbers prevails; some of their achievements have anticipated results of western scholars living in later centuries. It seems that besides astronomy and chronology, the requirements of the Vedic ritual are responsible for the ways on which the Hindu mathematicians of those days proceeded. The mathematics of Hellas starts from the philosophy of Pythagoras who claimed to have found the leading principle of the world order in the notion of number. On this foundation, the scholars of ancient Greece built up their mathematics which was the prototype of all theoretical sciences. At the present time mathematics is used all over the world for largely differing reasons. Everybody needs the "mathematics of the bazaar", a few are devoted to mathematics for its own sake, others apply it for research on astronomy, physics and other branches of science, a great number of people use it for engineering, navigation, survey, actuarial work, statistics and last not least for education. Let us postpone the last item; it will be discussed separately later. This short review furnishes as net result, that mathematics is mostly practised for some kind of application and not for its own sake. Thus we have to take into account the applicability of mathematics as one of its important features even if we refuse to derive its value from the value or non-value of those applications. It is a strange fact that mathematics whose habitat has been found in the human soul has so many applications in the outer world; it is one of those mysteries which we do not recognise as such because we got accustomed to them. Nevertheless we should try to give some kind of explanation.

Some clue could be found in Galileo's remark that the laws of nature are written in the language of mathematics, but this statement itself is made in the language of a particular philosophy and therefore it must be interpreted first. Laws are human institutions though the underlying principle of justice is of a higher nature. They are imperfect; their value is restricted by geographical and time limits and many offences are committed against them. However the laws given by the Creator to govern nature and supposed to be formulated in the language of mathematics, are in no perceptible connection with the idea of justice; they are eternal and universal; they cannot be broken; they are so perfect that not even room is left for an interference by the Almighty himself. Scientists have established several "laws of nature" and have expressed them in the language of mathematics. So they became able to explain the phenomena of nature and even to predict what will happen on earth and on the firmament whenever certain conditions are satisfied. Some scientists believed that only lack of sufficient mathematica, skill and incomplete knowledge of the physical data concerning the present state of nature prevented them from prestating all the future happenings in nature, since these were supposed to be uniquely determined by the laws of nature. However experience has confirmed those laws of nature found by the scientists only inside a certain sphere of phenomena; they have to be revised to cover a wider sphere and this procedure of alternating extension of our knowledge of physical facts and revision of the assumed laws of nature persists. Thus the laws found by the scientists are only approximations to the true laws of nature —— if there exist such laws. The statements of the physicists are made in the language of mathematics and we may suppose that the same will hold true for the statements to be made in future times, but even from this supposition one cannot conclude that the true laws can be announced in a mathematical form. The scientists of the present time are more modest in their aims than those of the 19th century. Neither do they pretend to know the laws given to nature by the Creator, nor do they expect to obtain this knowledge; they only attempt to lift a little bit the curtain which veils the mystery of nature. I suppose that most of the contemporary scientists when speaking of laws of nature, use this term only as a parable and so they do in many other cases. What seem to be fundamental notions of natural science, were known to the philosophers to be categories of human cognition, but the majority of the physicists were hardly conscious of the implications of this statement before the insufficiency of those

notions for a complete description of the happenings in nature became obvious. We lived to see that the categories of time, space, casuality can be applied to physics with limitations only and many other notions have been disclosed to be mere parables and images which are very helpful when used inside certain boundaries, but become useless outside of them. Theoretical science has reached a state where all the notions have become questionable and the mathematical method alone has remained permanent.

Thus it appears that in the human soul there is nothing so appropriate to help us for an understanding of the occurrences in the outer world as mathematics; it forms an important link between ourselves and the world around us. It would be a daring attempt to value the treasures of our soul and to put them into an order of merit, but certainly mathematics would not stand in the last place since it helps us to know something of the inner as well as of the outer world. This proves the importance of mathematics in the system of our knowledge and it shows that this science serves other purposes than the private enjoyment of a group of people, called mathematicians, who are obsessed by a cranky desire to investigate topics whose knowledge is of nobody's concern.

If one considers the high importance of mathematics, thus explained, one will understand why Plato did not admit anybody to his Academy who had not a fair knowledge of geometry. From that time dates the close interconnection between mathematics and education. It seems to be necessary to add a few words about this item because whenever the question "Why Mathematics?" is asked in public discussions, it concerns mathematics as a part of education, in particular of general higher education. That elementary mathematics is necessary for the requirements of daily life, is obvious, nor is there any serious objection against the training of experts in that subject by special courses. The debatable topic is whether some knowledge of mathematics above the level of elementary arithmetic is necessary for a well educated man. We should approach this question without any professional bias. Even Felix Klein has admitted that there are very talented persons who are incapable of following the ways of mathematical thinking and that these had better leave mathematics alone. One might however argue that just these people need more than others the strict discipline imposed by mathematical logic. One may expect that the rigorous form of mathematical conclusions will sharpen the critical sense and stiffen the student's mental selfdiscipline; moreover his experience that he

is able to check the correctness of mathematical investigations himself by logical considerations alone is likely to enhance his selfconfidence and the feeling of responsibility for his own thoughts. Dealing with mathematics is bound to kindle the flame of love for harmony, truth and spiritual beauty and to give strength to the intellect. If these considerations are correct, one must expect the professional mathematicians to be rare specimens of human perfection since they are living all their life under the purifying influence of their science. However looking at the biographies of eminent mathematicians — e. g. perusing Bell's "Men of Mathematics" — one will find that even this class of people are not free from short-comings of character. In the glaring light of biographical investigations some unpleasant features become visible which fortunately remain hidden to most of the contemporaries. Thus there is no wonder that my own impression about mathematicians is much more favourable. I like to remember the harmony and the spirit of good fellowship prevailing at the meetings of this Society at Lucknow, Hyderabad and Aligarh and the kindness shown by distinguished mathematicians of this country to me as a newcomer; moreover I feel much indebted to many mathematicians of other countries—living or deceased—to my teachers and to those with whom I used to exchange ideas. My general impression is that mathematicians, compared with other individuals of the same social classes, are amenable people, a rule which—as it has to be—is confirmed by very distinctive exceptions. However I have some doubt whether a few periods a week of mathematical teaching has a favourable influence on the character of the average student. As a matter of fact educationists are pointing more to the training of the intelligence by mathematics than to its influence on the character. So the controversy concerns the question: How far is mathematics helpful or necessary for the training of the intellect and how far should it be replaced by other means of instruction. A new element is coming into these discussions through the claims of several professions for a better and more specific mathematical training of their recruits and the suggestion that the foundations thereof should be laid by the general education. The complaints of the engineers are well known. Moreover I may mention that in public discussions arranged by the Calcutta Mathematical Society, a very distinguished American hygienist, strongly supported by two of his Indian colleagues, urged the necessity of some training in statistics for all the members of the medical profession. It is very probable that the same need will be felt soon for various other practical activities, e.g., general administration. To satisfy such claims, important changes in the system of

teaching are necessary, but these are meeting many difficulties. Unfortunately in many countries the curricula for teaching mathematics are still based more on tradition than on practical and educational considerations. Besides this outer difficulty, there is another one originating from the very nature of mathematics. It is impossible to form a curriculum simply by putting together those topics which are of particular practical or educational value. In mathematics everything is interdependent and every theory needs its proper foundation. Therefore a good syllabus must be a compromise considering many endevours and necessities; in every country every generation of mathematicians and educationists has to fulfill this task without being afraid of its difficulty. In mathematics first things must come first, whereas education wants a progress from the easy to the difficult; these two claims are often conflicting. For this reason the teaching of mathematics has its particular difficulties and it demands the highest standard of knowledge and educational skill from the teacher. Sceptics may ask whether the results justify such efforts: I personally believe that in every country better results could justify even much higher efforts. A detailed treatment of these educational questions as far as India is concerned must be left to the discussions during this meeting. Also at previous meetings, our Society has spent a considerable time on problems of mathematical education. These concern mathematics for education as well as education for mathematics. As mathematics has been proved to be an important branch of human activity, mathematicians have the duty to hand down their knowledge to succeeding generations and to make suitable educational arrangements for this purpose.

With these remarks on education, I may conclude my observations why mathematics is practised and why it must be practised. It has often been discussed whether it is an arts or a science subject; to many people it seems to stand on the border-line. I should not like to put it in this way. While arts-subjects are those concerned with the inner, and science-subjects those concerned with the outer world, mathematics is an arts-subject and simultaneously the basis of all the science-subjects with which it is connected in an indissoluble manner. So it stands as a symbol for the unity of the human mind.

St. Joseph's I. S. Press, Trichy.—1943.

THE ANNAMALAI UNIVERSITY

The Annamalai University was established more than fourteen years ago thanks to the generosity and public spirit of Rajah Sir Annamalai Chettiar of Chettinad whose name it bears. Its aims and ideals are similar to those of the other Universities in South India, but it differs from most of them in being both a teaching and a residential University. It differs from them in another respect also. It owes its existence to the munificence and generosity of an eminent citizen of India. His princely gift for the cause of higher education has no parallel in the history of this presidency.

Since the publication of the report of the Sadler Commission, the trend of the best educational opinion in the country has been in favour of a unitary, teaching and residential University, as it makes for efficiency and economy in higher academic work and provides ample facilities for the moulding of character and the development of a corporate spirit. To enable this ideal to be achieved for Tamil Nad, Rajah Sir Annamalai Chettiar offered to hand over the three Collegiate institutions for higher instruction in English, Tamil and Sanskrit founded by him at Chidambaram together with a princely donation of 20 lakhs of rupees. The Government showed their appreciation of this generous act by a contribution of 27 lakhs with a further sum of 7½ lakhs for buildings and an annual recurring grant which now stands at Rs. 1,90,000. The University began to function from July 1929. The Governor-General is the Visitor of the University; the Governor of Madras is the Chancellor; the Founder of the University is the Pro-Chancellor; the Vice-Chancellor is a whole-time officer who holds office for a period of three years and is appointed by the Chancellor from among three persons recommended by the Founder. At present we have in our midst as our Vice-Chancellor, M. Ruthnaswamy Esq., M.A., (Cantab) Barrister-at-Law, C.I.E.

About 150 miles from Madras, a stone's throw to the east of Chidambaram Railway Station lies the University in a suburb of the ancient and famous temple town of Chidambaram, called after the founder of the University, Annamalainagar. The campus of the University is 540 acres in extent, and the lecture halls, hostels, administrative buildings and residential quarters stand in this area. It is situated in the midst of green fields far from the bustle of

town life; and yet it has every amenity that most progressive towns enjoy. During these 14 years buildings costing more than 20 lakhs, have come up and great progress has been made in various directions. This is in no small measure due to the tireless efforts of the successive Vice-Chancellors, Sir S. E. Runganadhan (1929–35), now Adviser to the Secretary of State for India, the Rt. Hon. V. S. Srinivasa Sastriar (1935–39), Dr. Sir Kurma Venkata Reddi Naidu Garu (1940–42) and M. Ruthnaswami Esq., who has been the Vice-Chancellor since 1942.

In this University about 1200 students are pursuing to-day various courses of study in Arts, Sience, Oriental languages and literatures and in the conduct of research for the M.Litt., M.Sc., Ph.D., D.Sc. and D.Litt. degrees. A noteworthy feature is the Diploma course in South Indian Music leading after four years to the title of Sangita Bhushana, in which the student is taught vocal music or to play on the vina, violin, or mridangam. The University hopes to start soon courses in Technology, thanks to the generous donation of Rs. 5,00,001 recently made by Dr. K. V. Alagappa Chettiar.

The University Library is one of the best of its kind and is just a few yards from the hostel. It affords every facility for reading and contains many up-to-date books and periodicals. The study of current periodicals receives special encouragement. The total number of books in the general and sectional libraries is about 60,000. The Library was getting before the war about 250 scientific and learned periodicals. Each department has its own seminar library for the use of Honours and Research students.

The residential aspect of this unitary and teaching University deserves special mention. Students and teachers live in the university campus and there are many opportunities for fruitful contact of mind with mind. Every opportunity is utilized not only to develop the intelligence but also a sound character and a sound physique. The students' hostel is managed by a Warden and a Sub-Warden, and 8 resident tutors stay with the students to look after their life intellectual. The Warden is assisted in his work by a students' Representative Council. Living in close proximity to the teahers and the University Library, a student can consult without the least trouble any of his teachers about the books he should read, get his doubts cleared and discuss with them any other matter in which he may be interested.

The women students have a well-equipped hostel of their own under a lady Warden. In recent years more and more women students have joined the University, especially for the study of Music. Some have taken up advanced courses of study in Arts and Science.

The students capacity for organisation and leadership finds play in the largely self-governing University unions and societies. A Union Hall has been built at a cost of Rs. 15,000. The University has also a Dramatic Club, a Boating Club and a Students' Co-operative Society.

The University possesses one of the most extensive play grounds in South India and provision is made for all types of games and sports. In this University, Physical Training is compulsory for all students and no one can take a University Examination without producing a Certificate of physical training from the University Director of Physical Education. Particular care is taken of the health of the students. There is a well equipped University Hospital and the Resident Medical Officer periodically examines the students.

Ever since its inception the University has been keen on promoting research. In addition to the Research Journal, published three times every year, the University has to its credit a large number of learned publications. Studentships and fellowships have also been instituted for the promotion of advanced research. The departments of study are organized not merely for teaching but also for advanced research. Advantage has been taken of these facilities by students and by members of the teaching staff who have contributed many papers to learned journals.

Grants-in-Aid of research work are also awarded under certain conditions. With a view to providing for the advancement of Tamil a special research department is working under the guidance of the Head of the Department of Tamil. With a view to the ultimate adoption of Tamil as the medium of instruction the University has instituted prizes for text-books in Tamil on various subjects. So far books in Tamil on Logic, Physics and Chemistry have been published and others including one in Geometry are under publication.

Any casual visitor to Annamalainagar will be struck by the pulsating life of this centre of cultural activity. Any day of the week he will see in the evening hundreds of young men lustily playing Cricket, Tennis or Hockey or any one of the games popular with the youth of this country. Or he may see them sitting and silently

poring over books or periodicals in the spacious Reading Rooms of the University Library. Yet again he may see a scholar reading a learned paper before an advanced Study Circle of kindred spirits; or see him going out Scouting or on Social Service work. Perhaps he may light upon a Music performance given by one of the foremost exponents of Carnatic Music; or listen into the Radio or spend a few minutes reading a daily newspaper or current weekly. If he is more lucky he may see University professors, lecturers, students and townsmen gathered in the Srinivasa Sastri Hall listening to public lectures delivered by eminent visiting teachers and public men.

The Annamalai University is one of the youngest in the world. During the first decade of its existence it has served South India as a true centre of Indian culture and as a training ground for the young men and women of this land. Teachers and students cannot be sufficiently grateful to the Rajah of Chettinad for founding this magnificent institution which is unique in our country.

This young University considers it a great honour to welcome into its midst representatives of many of the older Universities and seeks their cooperation and help in the common task of the advancement of knowledge and the training of the young to take their proper place in the strenuous but formative times that lie ahead.

TRADITIONS ABOUT CHIDAMBARAM

1. Early Traditions

The great Sabhanayaka Shrine of Chidambaram (*par excellence,* the Koil), round which the ancient town has clustered, goes back to the earliest days of Saiva and Vaishnava reaction against the dominence of Jainism and Buddhism in the land. According to tradition, the temple was of divine origin, and its nucleus was divinely installed. Its most ancient votaries, Vyaghrapada (He of the tiger foot) and Patanjali (He of the serpent form)—held to be an incarnation of the thousand-headed Adisesha—form the earliest links in the religious traditions of the Shrine.

The legendary account of Patanjali is closely connected with the mystic dance of the Lord Siva, which taught a lesson to the proud Rishis of the Taruka forest, puffed up with conceit of Vedic learning (and held by a commentator to be followers of Mimamsa), and converted them into His fervent devotees. Vyaghrapada, the son of a Brahman hermit living on the Banks of the Ganges, was advised by his father to go to Tillai, a vast wilderness covered with trees of that name (*Excoecaria Agallocha*) and find the Parabrahma (Supreme Spirit) in that sacred spot. He bathed in the Sivaganga tirtham and worshipped a Lingam established under the shade of a banyan tree by its bank, whence arose the nucleus of the temple. Vyaghrapada soon found that he could not climb the lofty trees to get fresh flowers for his worship, prayed to the Lord and got the boon that his feet and hands might become those of a tiger, armed with strong claws and he be furnished with tiger's eyes, so that he might easily climb and see the flowers even before the dawn; he thus became known as 'the tiger-footed and six-eyed'; and a part of the present town came to be known as Tiru-Puliyur (Sacred Tiger Town); and also Perumbarrapuliyur.

Some time later, the great Adisesha assumed the form of half-man and half-serpent, in order to witness once again Siva performing the mystic dance in Tillai, and there met Vyaghrapada. He built a hermitage near by and installed a Lingam by a tank, which continue to this day as the Anantiswara Shrine and the Nagacheri tank, at the western end of the town.

A later legend relates to the Pallava King who was cured of leprosy by bathing in the Sivaganga tank and who changed his name of Simhavarman into Hiranyavarman (the golden-bodied) as a consequence. He repaired and added to the nucleus shrine. This royal patron has been equated with Simhavarman II, (or III) who reigned probably between 550 and 575 A.D. One of the later traditions refers to this king as a Chola (Stanza 12 of the *Kovilpuranam*). The picturesque story of his being commissioned by Vyaghrapada to guard Vyaghrapura and his being given the flag of the tiger-crest, is embodied in the *Koyilpuranam* of Umapathi Sivacharya, a Saivite scoliast of the later thirteenth and early fourteenth centuries. He is said to have belonged to the Dikshita community and his *samadhi* can even now be seen in Korravangudi (between the Chidambaram railway station and the University campus). Umapathi is the author of a good portion of the basic literature of *Saiva Siddhanta*, viz., the eight works beginning with the *Sivaprakasa*; he has also given a graphic account of the life of the great Seikkilar, the author of the *Periyapuranam*, (*Liber Sanctorum*) and of the work of Nambi-Andar Nambi in his two compositions, entitled "Sekkilar Nayanar Puranam" and "the Tirumurai-kandapuranam." The former of these two deals with an age when the memory of the great Chola rulers, Kulottunga I (acc. 1070) and Kulottunga II (acc. 1133), must have been relatively fresh in Umapathi's mind. Therefore, his account of Sekkilar, who was a contemporary of these kings, can be held to be much more historical in its perspective than the other work dealing with Nambi Andar Nambi, whose age can be fixed at the early eleventh century, if not at the close of the tenth.

Umapathi became the supreme theologian of the Tamil Siddhanta and is held to have learnt his wisdom from his master, Maraignana Sambanda, who belonged to a lower caste and was expelled from his own community for having partaken of food with his master. He forms the last of the quartette known as the *Samaya-Kuravar* and the *Santhana-Acharyas* of the *Saiva Siddhanta Darsana*. His *Koil-puranam* embodies legends which are, in point of time, long antecedent to those of the Saiva Nayanmars, described by Sekkilar and Nambi-Andar Nambi and may be said to constitute a portion of the oldest epoch of South Indian Saiva legends. Arumuga Navalar of Jaffna, a great Tamil scholar and Saiva pietist of the last century, edited the *Koilpuranam* and gave his own valuable commentary on the significance of the mystic dance of the Lord. According to one opinion, the comprehension of Patanjali among the earliest devotees

of Siva indicates the absorption in the Saiva teaching of the system of Yoga thought of which he was the founder.

II. Later Traditions

Most striking among the numerous associations of the 'Great Four' among the Saiva saints, is the legend connecting the temple with Manikkavacaga's victory over the Buddhists in the great disputation, described in the sixth canto of the *Tiruvadavurar Puranam*, which is an amplification of some sections of the *Madura Stalapuranam.* Therein we read how the great Saint of Vadavur was summoned from his retreat to confute the aggressive Buddhistic teachers who had come over to Chidambaram with the King of Ceylon at their head. The Chola King urged the saint to vanquish the Buddhistic disputants in an irrecoverable manner. Manikkavacaga vindicated the supreme power of Siva "seated as the Teacher in the shade of the beautiful banyan tree, teaching the laws of right", and as the Dancer in Tillai's beauteous golden hall with the girdle of the tiger skin (i. e. human passion) wrapped round him and wearing the serpent necklace (i. e. the guile and malice of mankind) and crushing under his foot the Demon Muyalagan, which is the embodiment of human depravity. This disputation might refer to the last epoch of the struggle of the Saivites with the Buddhists who continued to linger for a number of years in the isolated centres in the Tamil country, like Negapatam, and received frequent reinforcements from Ceylon. The legend can be attributed to about the ninth century.

Yet another attractive and heartening legend which has grown round the shrine is that of Nanda, the untouchable saint, who obtained final beatification in front of the Dancing Lord, becoming absorbed into him in a state of ecstatic devotion. The story of Nanda's piety is a very popular theme; and snatches of songs from the Nandanar Charitra Kirtana of Gopalakrishna Bharati are on everybody's lips in the Tamil country. The *kirtana* story has deviated in several respects from the version given in the *Periyapuranam.* The inclusion of Nanda among the traditional Sixty-three Saints should be regarded as being supplementary to his inclusion in a decade of Sundaramurthi Nayanar's *Tevaram*, which can be ascribed to the first quarter of the ninth century A.D. The legend is certainly older than that date. Stressing the great moral value of this legend, one may repeat the words of Sir Willim Wilson Hunter, when extolling the shrine of Jagannath, that as long as the towers of

Chidambaram exist, so long will there be in the land "the perpetual and visible protest of the equality of man before God."

The village of *Tiruvetkulam* by the side of which the Annamalai University Settlement has been located is credited by local tradition with having been the place of Arjuna's penance, where God Siva presented the *Pasupatastram* to the pandava hero. Its temple dedicated to Pasupatiswara was renovated about a quarter of a century ago by Diwan Bahadur S. Rm. Ramaswami Chettiar, an elder brother of the Rajah Saheb of Chettinad. It contains an epigraph dated Saka 1488, recording the grant of the village by Achyutappa Nayak of Tanjore for the merit of his overlord King Tirumala I of Vijayanagar. The temple has been sung by the great Saiva Saints, Appar and Gnanasambandar. The latter refers to the village as being situated near the sea, where the chanting of the Vedas and the performance of the Vedic sacrifices went on; and the former describes the Shrine as the abode of the Vetanar, i. e., the Hunter, Siva as Kirata. It is held by learned opinion that the image of Kiratarjunamurti in this temple shows unmistakable affinity with Pallava stone sculptures of the seventh century A. D., and there is reason to believe that it is the same image that Appar saw and the presence of which led him to celebrate the temple in his hymns as the abode of Vetanar. There are two other images of remarkable value and antiquity in the temple, namely, those of Arjuna and Parvathi. The image of Arjuna is later, but may not be far removed in point of time from that of the Kiratarjunamurti.

CHIDAMBARAM AND ITS GREAT NATARAJA SHRINE

Chidambaram is famous for the great temple of Sri Nataraja, one of the most venerated Saiva Shrines of South India. The temple contains also the well-known and ancient Vaishnava Shrine of Sri Govindaraja, and is one of the largest in South India and covers an area of about 39 acres.

The Walls

It is surrounded by two high walls faced with dressed granite—the outer of which is believed to be due to Virappa Naick (1572-95) of Madura who made improvements also in the Trichinopoly Fort, in the fort at Aruppukkottai and the great Minakshi shrine of Madura. Both the enclosing walls are about 30 feet high; and the outer is faced with stone on both sides throughout. It is these walls that enabled the conversion of the temple into a fort during the Carnatic and Mysore Wars of the 18th century and made it the scene of some fighting. Outside the Temple run the present four car streets about 60 feets wide, said to cover the site formerly occupied by the ditch which once formed part of the fortifications.

The Gopurams

The inner of the two walls is pierced on each of the four sides by a great Gopuram, the lower part of which is of stone, not elaborately sculptured; and the upper portion is of brick and mortar profusely ornamented with figures of men and objects of Puranic mythology.

The northern Gopuram was built by Krishnadeva Raya, the great ruler of Vijayanagar (1509-1530 A. D.). The southern Gopuram bears inscriptions of Sundara Pandya who lived about 1250 A. D. This last was repaired in the latter part of the 18th century by the well-known Pachaiyappa Mudaliar whose benefactions are spread throughout South India and in whose name a Choultry and an English high School are maintained, at the place besides several *Kattalais* in the temple. The east Gopuram bears representations of Pachaiyappa and his wife; and the agraharam outside it, known as the East Sannadhi, was also founded by him. On all the Gopurams, the most common among the sculptured panels of stone are the dancing figures—symbolical of the central idea of the temple, *viz.* the mystic dance of Siva who appears here as Nataraja, the God of dance.

The Origin of the Temple

The central core of the temple certainly goes back to Pallava times if not to earlier epochs. According to legend, it was founded by one Svetavarman who was cured of his leprosy by bathing in the Sivaganga Tank and became transformed into Hiranyavarma Chakravarthi (the golden-bodied emperor). Even before his time the Dikshitars, who are a small endogamous community of temple-priests and tie their top-knots of hair to the side of their foreheads and who manage the temple which they regard as their property, are supposed to have come from Benares to the number of 3000 (they are now only about a couple of hundred families or less). When arrived at the shrine, one of the three thousand was missing and when a search was made the Lord Siva himself came down as the missing one. They claim for themselves a divine origin and declare that Brahma took them from Tillai—(Chidambaram is known as *Tillai-vanam* or Tillai-forest)—to Benares to perform a Yaga; and they strayed at the latter place till Hiranyavarma, at the instance of the deity, called them back.

Its management

All the married Dikshitars (and they marry early) have a voice in the management of the temple; and a certain number of them are always on duty at the temple; and they periodically meet in the Deva Sabha—one of the five Sabhas of the Shrine—to transact business, when a Pandaram brings a lamp from the Nataraja Shrine and acts as the convener of the meeting, to preserve the appearance of the perfect equality among the Dikshitars. Each has his own particular clientele; and all share in the general perquisites of the Shrine.

Its Growth

The Thevaram hymners mention the existence of the Kanaka Sabha (the golden hall), ever the most important Shrine and the abode of Nataraja. Parantaka Chola, one of the great Chola sovereigns of Tanjore of the 10th century, is recorded to have covered this shrine with plates of gold; and Vikrama Chola who ruled in the 12th century claims that he covered a considerable portion of it with gold. The great poet-saint, Manikkavachakar who is put in the 8th century by G. U. Pope frequently sings of the glory of *Chittambalam* (the Shrine of wisdom) of which Chidambaram is a variant, and of *Ponnambalam* (the Shrine of gold), and of the Lord Siva who dances here. He sings: "Him have I seen in Tillai's court where worships all the world."

Manikka Vachakar

The saint's whole life may be regarded as one continuous glorification of Chidambaram; and the many hymns that he composed of the God of Tillai are extremely beautiful. Here the saint won his greatest triumph by refuting in argument the Ceylon Buddhists who had come to overthrow the Saiva cult. "His hymn connected with his victory is even now sung in the temple on the anniversary of the event. It was here also that his hymns were taken down from his lips; and once a year the god is taken to the spot where this was accomplished. Finally it was within the shrine of which he had sung so often that he at length obtained final beatitude" (Francis: *South Arcot District Gazetteer* 1906, p. 274).

Saint Nandanar

It was also at Chidambaram that the great untouchable saint Nandanar, worshipped and obtained his final beatitude; and the tank (known as the Homakulam) where he bathed before he performed the rites which converted him to saint-hood, is to be seen to the south-west of the temple.

The Amman Shrine

The enclosed buildings in the shrine are numerous and though they have grown unconsciously and in very many stages and are not part of any one general scheme, their general effect is striking enough. The Amman Shrine (sacred to Sivakamasundari) is a beautiful structure, remarkable for its porch which, according to the high authority of Fergusson, is 'of singular elegance' and whose roof is supported by a peculiar and skilfully arranged system of "vaulting or rather bracketing shafts." The whole shrine, though modernised still preserves its ancient elegance and is picturesquely enclosed in a paved court surrounded by a gallery supported by graceful columns.

The Shrine of Subrahmanya

North of this Shrine is the temple of Subrahmanya known as the Pandyanayakan Kovil, which has recently been repaired and is built on a high basement whose sides are beautifully sculptured, the roof being supported by a number of finely worked pillars. The sacred Sivaganga tank is surrounded on all sides with stone-steps and by a beautiful pillared cloister recently renovated.

The Raja Sabha

To the east of it is the famous 1000-pillared Mantapa—often called the *Raja-Sabha* or the Hall of State—a forest of granite pillars, with a series of brick-vaulted halls running down the middle and passing through magnificent stone steps to the courtyard on the south. A number of slender granite columns dot this courtyard and serve as the posts for the big festival pandal which forms a continuation of the hall. It was in this hall that many of the Pandya and Chola rulers crowned themselves on their return from victorious campaigns.

The inner enclosure and the Shrine

The innermost enclosure to the south of the tank contains 4 out of the 5 *Sabhas* of which the Shrine is composed; *viz* the *Deva-sabha*, the *Chit-sabha*, the *Kanaka-sabha* i.e. the porch in front of the idol and the *Nritta-sabha*. The last, situated close to the southern wall of the inner enclosure and facing the Kanaka Sabha (and now sadly mutilated on the sides) is a most beautiful structure—a celestial vehicle of a mantapam adorned with finely-carved wheels and horses on either side. The stylobate is ornamented with dancing figures; and the domed roof is supported on finely-carved pillars; and Fergusson speaks of those carvings as "more graceful and more elegantly executed than any others of their class, so far as I know, in South India." A modern cloister of great width runs round the walls of the inner enclosure and will be complete but for the projection of the Vaishnava Amman Shrine which stands in the south-west corner. In the *Chit-sabha*, is supposed to be enshrined the *Akasa Linga* (ether linga) one of the five "lingams of the elements" in South India. This is symbolised in what is known as the Chidambara *Rahasya* indicated by a small space behind the idol, faced with *chakra*-stones and a long string of golden *bilva*-leaves.

The Mulasthana

Near the Deva Sabha and situated between the wall enclosing the Chit-Sabha and the inner enclosure wall is the *Mulasthana* shrine, whose walls are known by the name of Vikrama-Cholan Thirumaligai. Up to the time of this king, the transactions of the temple were in the name of Chandesvara; and the image of Siva-linga in the Mulasthanam was long considered to be the *sanctum sanctorem.* The prominence of the Nataraja shrine was probably of later growth, as is further indicated by the provision made by the Cholas

for the singing of the Thevaram hymns only in the Mulasthanam (Inscription No. 341 of 1913—in the Govt. Epigraphist's Report).

The Vaishnava Shrine

The Vaishnava Shrine in the inner enclosure is dedicated to Govindaraja, in his form of Vishnu sleeping on the Serpent. It is a sacred spot for Vaishnavas and has been praised by some of the Alvars. This Shrine was brought into great prominence under the later Vijayanagara kings, having suffered an earlier eclipse under the Cholas.

The Sthala Purana

The Sthala Puranam of the Shrine is entitled the *Koyil Puranam*—as *Koyil* is the name applied to the Chidambaram Shrine. This Puranam is attributed to Umapathi Sivacharya. In the Purana we are told of Vyaghrapada and of Patanjali both of whom planted a hermitage and enshrined a lingam; by-and-by they got up the great festival (Arudra-Darsanam) coming off on the Tiruvathurai Nakshatra in the Tamil month of Margali which is still observed as the grandest festival of the district. The stories of Patanjali and Vyagrapada are possibly legends "to link on the Saiva system of the south with Vedic times." And the Story of *Tillai Kali* having been shamed by Siva's dance into leaving the place—her temple still stands outside the bounds—is symbolical of the eclipse of the previous popular cult by Saivism.

GINGEE

The Fort of Gingee

Situated about 18 miles from the Tindivanam station, which is midway between Madras and Chidambaram lie the ruins of the immense fortifications of Gingee embracing 3 hills which form the angular points of a roughly equilateral triangle with a huge rampart and a ditch. These outer walls enclosing the hills are even now in a state of good preservation, being built of granite. The three hills enclosed in the outer fort form three citadels. The one situated on the west called Rajagiri (originally called Kamalagiri or Anandagiri) is the most impregnable, being nearly 800 feet in height. Just about the middle the hill shoots up at its northern end into a perpendicular eminence, cut off from all communication by a deep chasm about 10 yards wide over which there is a small wooden bridge now. The hill has embrasured walls and gateways built along all its precipitous shelves and edges; and it appears to the observer to be one mass of fortifications rising tier above tier in wild picturequeness.

The Lower Fort

At the base of Rajagiri is what might be called the inner citadel (or Lower Fort) equipped with walls and bastioned gateways, and other strategic requirements. In this are situated two pagodas dedicated to the Gods Venkataramana and Pattabhirama, which before their destruction by the French, were adorned with ornamented and sculptured walls and carved pillars many of which were taken away to Pondicherry. The pillars round the base of the statue of Dupleix in the Place de la Republique in Pondicherry once belonged to this place. The Jains of Sittamur, a village in the vicinity, are said to have carried away from these a stone-car for their own use. There are also two huge brick-granaries of great dimensions; and the visitor will find that the least noise made in them echoes tremendously. The Pattabhiramaswami Temple is architecturally worth preserving, particularly the 12-pillard mantapam in front of the shrine. There are two tanks in the inner fort lying at diffreent levels, named Chettikulam and Chakrakulam. At a corner of the embankment of the former, on its north side, is the masonry platform on which it is said that the body of the heroic Desing Raja (the subject of the

well-known Ballad of Gingee) was burnt and his young wife committed *sati*. Above the other tank which is a little higher up is a low circular brick parapet, called the 'prisoners well' into which prisoners were said to have been thrown and left to die of starvation. There are also in the fort 2 great slabs of polished stone, called the Raja's and the Rani's bathing stones. Nearby is the Kalyana Mahal, perhaps the most curious building in the place. It consists of a rectangular court surrounded by rooms, said to have been used by the ladies of the Raja's or the governor's household. In the middle of one side of the court rises a square tower of eight stories, built of stone plastered over and having a roof pyramidal in shape. The plan of each of the storeys is the same, being a single room, about 8 feet square surrounded by a verandah which is supported on arches. This is the most prominent building in all the lower fort. It has got several curious pieces of earthen-ware pipe built into the walls, probably for carrying water down from the upper storeys.

Rajagiri Hill

The walk up the flight of steps to the summit of Rajagiri is very pleasant, the grey granite steps leading through thick green jungle. On the plateau of the hill, under the shadow of the steep summit, is a grove of tamarind trees in which there is a temple to Kamala Kanni Ammal, the *genius loci* of the place, to which buffaloes are periodically offered as sacrifice. The temple has been standing from very ancient times. On the top of the steep summit, divided from the path leading to it by a chasm, is a fairly level space containing a shrine of God Ranganatha and a mantapam, two big masonry granaries, a masonry flag-staff, a structure of very thick walls, intended possibly as a magazine, and another chamber, known as the treasury. There are two springs of water in the rock which never run dry.

Krishnagiri and other places

Krishnagiri, the next in importance of the hills, is on the north of the place, and contains two stone-built granaries, a mantapam, a temple to God Ranganatha who was the patron deity of Raja Desing, and a curious edifice with a domed roof, known as the audience-chamber, and containing arcades on pointed arches built in the Indo-Muhammadan style. Chandrayan Drug, the third hill on the south, is the least important and contains now nothing worth mention except an empty mantapam.

All the buildings in the Gingee Fort are now conserved by Government and many of them have been repaired.

The origin of the name

The origin of the name of the place is obscure; the name Senji is derived variously from Sanjivi, the panacea of Hindu mythology, and from Singavaram, a neighbouring rock-cut Vaishnava shrine whose god is supposed to be the chief deity of the locality. According to legend, a virgin Senjiamman, committed suicide, along with her six virgin sisters, because they were threatened with a violation of their chastity; their spirits are even now believed to be haunting the place; and each of them has got her own little shrine. Senjiamman who is worshipped on one of the hills gave her name to this particular rock; and this name afterwards came to be applied to the whole place, including the hills and the village below. One other of the sisters, Kamalakanni Amman is worshipped in a small Shrine on Rajagiri, as noted above.

The history of the place

The early history of the place is very difficult to construct. It is not probably earlier than the 11th century when a feudatory of the Chola was ruling at the town of Singavaram in the neighbourhood. During the period of Chola decay in the 13th century Gingee should have arisen as an independent principality. At first it was ruled by a dynâsty of shepherd chiefs, known as *kons*. Then the place came under the rule of the Rajas of Vijayanagara, one of whose early chiefs, the Brahman Gopanarya, exercised sway from this place, and recovered from the Mussalmans the image of God Ranganatha and removed it from Tirupathi to Singavaram. After keeping it for some time, he is said to have taken the godback to its temple at Srirangam and reconsecrated it.

The Naick chiefs, also feudatories of Vijayanagar, ruled over Gingee for a long time from about the beginning of the 16th century. The construction of many of the buildings both in the upper and the lower forts should be ascribed to them; and they were besides great patrons of learning. It subsequently came under the control of the Bijapur Sultan and subsequently under that of the celebrated Sivaji. The Marathas strengthened the place very much and used it as their headquarters when they had to fight Aurangzib away from their own native land.

After an exhausting 8-years-long siege, the Mughals were able to wrest the fortress from the Marathas in 1698. A few years later Aurangzib granted Gingee as a fief to a Bundela chief named Sarup Singh. His son, Desing, was head-strong and short tempered provoked a quarrel with the Nawab of the Carnatic and died bravely fighting against tremendous odds. Even now wandering minstrels sing to groups of simple villagers gathered under the banyan-tree, of the heroism of Desing of Gingee, of how he loved and fought and fell; as well as of his brave *Sati* queen. Later Gingee was captured by the French General Bussy by a brilliant attack, only to be taken along with the other forts of the Carnatic by the English in 1760. But though freed from the ravages of war, the place became a prey to malaria, and was rapidly deserted and depopulated. It has however begun to revive in recent years and has been made the head-quarters of a taluk.

The historical fortunes of Gingee

The fortunes of Gingee as they developed through the centuries are illustrative of the vicissitudes through which this part of the Carnatic passed, first emerging under indigenous tribal occupation, then coming under Chola rule and under the sway of Vijayanagar and of its Nayak governors, subsequently serving as the battle-field of Bijapurian and Kutb Shahi ambitions in the Carnatic, also utilised by Shahji and Shivaji with a prevision that is certainly astonishing as a possible southern base for future Maratha resistance to the Mussalmans, consequently experiencing a long-drawn-out siege by Aurangzib's forces, and after its incorporation into Mughal dominion, flourishing for a time as the head-quarters of the Mughal subah of the Carnatic.

Next, the emergence of the brilliant, but short-lived and misdirected chivalry of Raja Desing, the well-known hero of ballad and legend; and finally the appearance on the scene, of the French under the famous soldier, Bussy, the consequent exposure of the fortress to the violent interplay of the political forces that indulged in their 'Witches Revel' in the decades that saw the birth of British Supremacy, during which some stirring figures passed over the scene; e. g., Bussy, the ill-fated Nasir Jang and the adventurous Hyder Ali. The architectural and monumental glories of Gingee are even now intact, in substantial volume and preservation and claim for it a rank among the largest and most striking of the historic fortresses of our land.

OTHER PLACES OF INTEREST IN SOUTH INDIA

Madura

Madura is an ancient city of hoary traditions and has been the most famous and prosperous town in the Tamil Country from the centuries before the Christian era. It was the capital of the Pandyas and one of the chief trading centres from which pearls, cotton goods, spices etc., were exported to the Roman Empire.

It was the seat of the famous Academy known as the Tamil *Sangham* at which was sung the immortal kural of Tiruvalluvar and other Tamil classics. Tradition says that in far-off times when the land stretched a great number of leagues to the south of Cape-Camorin, Madura was the seat of two other flourishing sanghams. It is also the seat of many of the legends about the acts of Siva embodied in the *Thiruvilaiyadal puranam.*

Apart from being a great centre of learning, Madura was in the ancient days conspicuous for its fort with its 4 great towers hence it was known as *Nan-Madakkudal* (Cluster of 4 Towers) shortened into Kudal. The site of this ancient Madura is now in ruins and supposed to lie some six miles to the south east of the present city. West of this old city was the hill known as Parankunru, sacred to God Subrahmanya and sung by the poet Nakkirar. It is now to the south-west of the modern City.

The Pandya rulers flourished till about the 10th century when they were reduced to subjection by the Cholas of Tanjore. Later they came to eminence in the 13th century, only to fall under the sway of the Mussalman invaders from Delhi who advanced under the infamous general Malik Kafur who planted a Mussalman garrison in the city. The Mussalman Governor of Madura quickly asserted his independence, and the Sultanate of Madura flourished for over half a century till it was destroyed by the Rayas of Vijayanagar who were champions of the cause of Hinduism. The Mussalmans subjected the city to great destruction and the people to oppressive cruelty; they pulled down its outer walls and towers, destroyed many streets and buildings and only spared the great temple of Minakshi and Sundaresvara.

The Nayaks were great builders and largely extended and improved the temple dedicated to Minakshi, an ancient princess who married Siva himself. The Temple is the most ornate and famous in the country and is embodied in numerous legends. The Hall of 1000 Pillars is almost regular and has got four fine and tall gopuras and five smaller ones. Its beautiful small tank, known as the Golden Lily Tank, is surrounded by well-sculptured arcades and the Hall of 1000 columns has sculptures which surpass those of any other hall of this class in marvellous and finely conceived elaboration of design. The Kalyana Mantapa, said to have been built in the beginning of the 18th century is also a fine piece of art. The total effect is very imposing and makes the temple more remarkable than any other of its kind. The Hall of the 1000 pillars was erected by Aryanatha Mudali, the general of Visvanatha Nayak. The groups of figures, carved from single huge stones in this Hall and elsewhere in the Temple are wonderful works of the sculptor's art.

Many of the important buildings of Madura are the creation of the famous Tirumala Nayak who ruled from 1623 to 1659. In his time were built the *Pudumantapa* or *Vasanta mantapa* in front of the Temple as a sort of a summer retreat for the God. It is a grand hall nearly 350 feet in length and over 100 feet wide, resting on four parallel rows of pillars each one of which is elaborately decorated with sculptures different in design from the others.

The Tamkam, situated on the northern bank of the Vaigai, and now the official residence of the Collector of Madura, was another creation of Tirumala Nayak. It is a curious building of semi Moorish architecture and was intended to serve as a pleasure-house from which to view races and combats.

Madura has always been the centre of religious and educational activity. It was, as we know, the seat of the ancient Tamil Academy, where flourished men of letters and poets belonging to various religious persuasions. Jainism and Buddhism flourished long in the land till the epoch of the Siva Nayanmars, of whom Sambandar distinguished himself greatly by converting the great king Kun Pandyan (7th century) from the Jaina to the Saiva fold and followed it up by a persecution of Jaina religious teachers. After this royal conversion the Jaina faith quickly disappeared from the pandya country and the Tamil land. The Buddhist faith suffered a similar fate; but relics of it are seen in a few caves cut out of the hill sides in the District which contain curious old inscriptions in the Brahmi script.

The city is even now one of the chief centres of educational and religious activity in South India. The Madura Tamil Sangham started by the munificence of the Rajas of Ramnad and other Zamindars tries to cultivate Tamil scholarship of a high kind and resuscitate old Tamil learning.

The *European missionaries* have always had in Madura one of their chief centres of religious activity. From the time of the early Nayaks the Jesuits had a Mission centre here associated with the great names of Robert De Nobili, and Constantius Beschi who did much for the spread of Christianity in the land.

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